NAG Library Function Document

nag_tanh (s10aac)

1 Purpose

nag_tanh (s10aac) returns a value for the hyperbolic tangent, $\tanh x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_tanh (double x)
```

3 Description

nag_tanh (s10aac) calculates an approximate value for the hyperbolic tangent of its argument, $\tanh x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x\sum_{r=0} a_r T_r(t)$$

where $-1 \leq x \leq 1, \quad -1 \leq t \leq 1, \quad$ and $\quad t = 2x^2 - 1.$

For $1 < |x| < E_1$ (see the Users' Note for your implementation for value of E_1)

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For $|x| \geq E_1$, $\tanh x = \text{sign}\, x$ to within the representation accuracy of the machine and so this approximation is used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left|\frac{2x}{\sinh 2x}\delta\right|.$$

That is, a relative error in the argument, x, is amplified by a factor approximately $\dfrac{2x}{\sinh 2x}$, in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

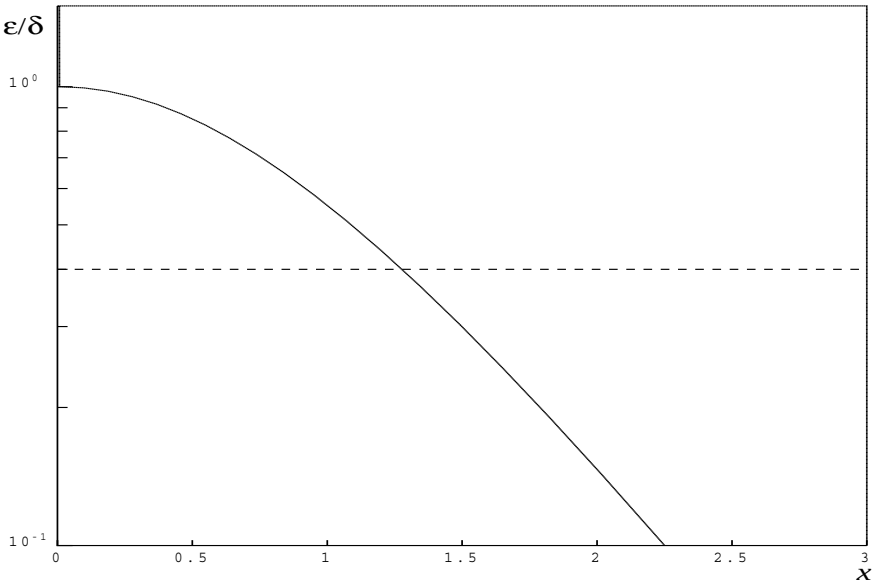

Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from $x = 0$ the accuracy will eventually be limited entirely by the precision of machine representation.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_tanh (s10aac) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer exit_status = 0;
  double  x, y;
```

```
  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_tanh (s10aac) Example Program Results\n");
  printf("           x       y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
    {
      /* nag_tanh (s10aac).
       * Hyperbolic tangent, tanh x
       */
      y = nag_tanh(x);
      printf("%12.1f%12.5f\n", x, y);
    }

  return exit_status;
}
```

10.2 Program Data

```
nag_tanh (s10aac) Example Program Data
           -20.0
            -5.0
             0.5
             5.0
```

10.3 Program Results

```
nag_tanh (s10aac) Example Program Results
           x       y
       -20.0    -1.00000
        -5.0    -0.99991
         0.5     0.46212
         5.0     0.99991
```