NAG Library Function Document

nag_deviates_studentized_range (g01fmc)

1 Purpose

nag_deviates_studentized_range (g01fmc) returns the deviate associated with the lower tail probability of the distribution of the Studentized range statistic.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_studentized_range (double p, double v, Integer ir,
       NagError *fail)
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups, can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left(r \int_{-\infty}^{\infty} \phi(y)(\Phi(y) - \Phi(y - qx))^{r-1}\, dy \right) dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

For a given probability p_0, the deviate q_0 is found as the solution to the equation

$$P(q_0; v, r) = p_0, \tag{1}$$

using a root-finding procedure. Initial estimates are found using the approximation given in Lund and Lund (1983) and a simple search procedure.

4 References

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32(2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Arguments

1: **p** – double *Input*

On entry: the lower tail probability for the Studentized range statistic, p_0.

Constraint: $0.0 < \mathbf{p} < 1.0$.

2: **v** – double *Input*

On entry: v, the number of degrees of freedom.

Constraint: $\mathbf{v} \geq 1.0$.

3: **ir** – Integer *Input*

On entry: r, the number of groups.

Constraint: $\mathbf{ir} \geq 2$.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ACCURACY

Warning – There is some doubt as to whether full accuracy has been achieved.

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_INIT_ESTIMATE

Unable to find initial estimate.

NE_INT

On entry, $\mathbf{ir} = \langle value \rangle$.
Constraint: $\mathbf{ir} \geq 2$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $0.0 < \mathbf{p} < 1.0$.

On entry, $\mathbf{v} = \langle value \rangle$.
Constraint: $\mathbf{v} \geq 1.0$.

7 Accuracy

The returned solution, q_*, to equation (1) is determined so that at least one of the following criteria apply.

(a) $|P(q_*;v,r) - p_0| \leq 0.000005$

(b) $|q_0 - q_*| \leq 0.000005 \times \max(1.0, |q_*|)$.

8 Parallelism and Performance

Not applicable.

9 Further Comments

To obtain the factors for Duncan's multiple-range test, equation (1) has to be solved for p_1, where $p_1 = p_0^{r-1}$, so on input **p** should be set to p_0^{r-1}.

10 Example

Three values of p, ν and r are read in and the Studentized range deviates or quantiles are computed and printed.

10.1 Program Text

```
/* nag_deviates_studentized_range (g01fmc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{

  /* Scalars */
  double   p, v, valq;
  Integer  exit_status, i__, ir;
  NagError fail;

  exit_status = 0;

  INIT_FAIL(fail);

  printf(
          "nag_deviates_studentized_range (g01fmc) Example Program Results\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
```

```
#endif

  printf("\n%s\n\n", "  p     v    ir    Quantile ");
  for (i__ = 1; i__ <= 3; ++i__)
    {
#ifdef _WIN32
      scanf_s("%lf%lf%"NAG_IFMT"%*[^\n] ", &p, &v, &ir);
#else
      scanf("%lf%lf%"NAG_IFMT"%*[^\n] ", &p, &v, &ir);
#endif

      /* nag_deviates_studentized_range (g01fmc).
       * Computes deviates for the Studentized range statistic
       */
      valq = nag_deviates_studentized_range(p, v, ir, &fail);
      if (fail.code == NE_NOERROR || fail.code == NE_ACCURACY)
        {
          printf("%5.2f%2s%4.1f%1s%3"NAG_IFMT"%1s%10.4f\n", p, "", v,
                  "", ir, "", valq);
        }
      else
        {
          printf(
                  "Error from nag_deviates_studentized_range (g01fmc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_deviates_studentized_range (g01fmc) Example Program Data
0.95 10.0  5
0.3  60.0 12
0.9   5.0  4
```

10.3 Program Results

```
nag_deviates_studentized_range (g01fmc) Example Program Results

  p     v    ir    Quantile

 0.95  10.0   5     4.6543
 0.30  60.0  12     2.8099
 0.90   5.0   4     4.2636
```