NAG Library Function Document

nag_deviates_chi_sq (g01fcc)

1 Purpose

nag_deviates_chi_sq (g01fcc) returns the deviate associated with the given lower tail probability of the χ^2-distribution with real degrees of freedom.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_chi_sq (double p, double df, NagError *fail)
```

3 Description

The deviate, x_p, associated with the lower tail probability p of the χ^2-distribution with ν degrees of freedom is defined as the solution to

$$P\left(X \leq x_p : \nu\right) = p = \frac{1}{2^{\nu/2}\Gamma(\nu/2)}\int_0^{x_p} e^{-X/2}X^{v/2-1}\,dX, \quad 0 \leq x_p < \infty; \nu > 0.$$

The required x_p is found by using the relationship between a χ^2-distribution and a gamma distribution, i.e., a χ^2-distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

For very large values of ν, greater than 10^5, Wilson and Hilferty's normal approximation to the χ^2 is used; see Kendall and Stuart (1969).

4 References

Best D J and Roberts D E (1975) Algorithm AS 91. The percentage points of the χ^2 distribution *Appl. Statist.* **24** 385–388

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

5 Arguments

1: **p** – double *Input*

On entry: p, the lower tail probability from the required χ^2-distribution.

Constraint: $0.0 \leq \mathbf{p} < 1.0$.

2: **df** – double *Input*

On entry: ν, the degrees of freedom of the χ^2-distribution.

Constraint: $\mathbf{df} > 0.0$.

3: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

On any of the error conditions listed below except **fail.code** = NE_ALG_NOT_CONV nag_deviates_chi_sq (g01fcc) returns 0.0.

NE_ALG_NOT_CONV

The algorithm has failed to converge in ⟨*value*⟩ iterations. The result should be a reasonable approximation.

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_GAM_NOT_CONV

The series used to calculate the gamma function has failed to converge. This is an unlikely error exit.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_PROBAB_CLOSE_TO_TAIL

The probability is too close to 0.0 or 1.0.

NE_REAL_ARG_GE

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{p} < 1.0$.

NE_REAL_ARG_LE

On entry, $\mathbf{df} = \langle value \rangle$.
Constraint: $\mathbf{df} > 0.0$.

NE_REAL_ARG_LT

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{p} \geq 0.0$.

7 Accuracy

The results should be accurate to five significant digits for most argument values. Some accuracy is lost for p close to 0.0.

8 Parallelism and Performance

Not applicable.

9 Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to nag_deviates_gamma_dist (g01ffc) made.

10 Example

This example reads lower tail probabilities for several χ^2-distributions, and calculates and prints the corresponding deviates until the end of data is reached.

10.1 Program Text

```
/* nag_deviates_chi_sq (g01fcc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
  Integer  exit_status = 0;
  double   df, p, x;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_deviates_chi_sq (g01fcc) Example Program Results\n");
  printf("     p       df      x\n\n");
#ifdef _WIN32
  while (scanf_s("%lf %lf", &p, &df) != EOF)
#else
  while (scanf("%lf %lf", &p, &df) != EOF)
#endif
    {
      /* nag_deviates_chi_sq (g01fcc).
       * Deviates for the chi^2 distribution
       */
      x = nag_deviates_chi_sq(p, df, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_deviates_chi_sq (g01fcc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%8.3f%8.3f%8.3f\n", p, df, x);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_deviates_chi_sq (g01fcc) Example Program Data
0.0100 20.0
0.4279 7.50
0.8694 45.0
```

10.3 Program Results

```
nag_deviates_chi_sq (g01fcc) Example Program Results
     p       df      x

   0.010  20.000   8.260
   0.428   7.500   6.200
   0.869  45.000  55.759
```