

# NAG Library Function Document

## nag\_dgglse (f08zac)

### 1 Purpose

nag\_dgglse (f08zac) solves a real linear equality-constrained least squares problem.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_dgglse (Nag_OrderType order, Integer m, Integer n, Integer p,
                double a[], Integer pda, double b[], Integer pdb, double c[],
                double d[], double x[], NagError *fail)
```

### 3 Description

nag\_dgglse (f08zac) solves the real linear equality-constrained least squares (LSE) problem

$$\underset{x}{\text{minimize}} \|c - Ax\|_2 \quad \text{subject to} \quad Bx = d$$

where  $A$  is an  $m$  by  $n$  matrix,  $B$  is a  $p$  by  $n$  matrix,  $c$  is an  $m$  element vector and  $d$  is a  $p$  element vector. It is assumed that  $p \leq n \leq m + p$ ,  $\text{rank}(B) = p$  and  $\text{rank}(E) = n$ , where  $E = \begin{pmatrix} A \\ B \end{pmatrix}$ . These conditions ensure that the LSE problem has a unique solution, which is obtained using a generalized  $RQ$  factorization of the matrices  $B$  and  $A$ .

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia

Anderson E, Bai Z and Dongarra J (1992) Generalized  $QR$  factorization and its applications *Linear Algebra Appl. (Volume 162–164)* 243–271

Eldèn L (1980) Perturbation theory for the least squares problem with linear equality constraints *SIAM J. Numer. Anal.* **17** 338–350

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **m** – Integer *Input*

*On entry:*  $m$ , the number of rows of the matrix  $A$ .

*Constraint:*  $m \geq 0$ .

- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrices  $A$  and  $B$ .  
*Constraint:*  $n \geq 0$ .
- 4: **p** – Integer *Input*  
*On entry:*  $p$ , the number of rows of the matrix  $B$ .  
*Constraint:*  $0 \leq p \leq n \leq m + p$ .
- 5: **a**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  
 $\max(1, \mathbf{pda} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pda})$  when **order** = Nag\_RowMajor.  
The ( $i, j$ )th element of the matrix  $A$  is stored in  
 $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$  when **order** = Nag\_RowMajor.  
*On entry:* the  $m$  by  $n$  matrix  $A$ .  
*On exit:* **a** is overwritten.
- 6: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **a**.  
*Constraints:*  
if **order** = Nag\_ColMajor,  $\mathbf{pda} \geq \max(1, \mathbf{m})$ ;  
if **order** = Nag\_RowMajor,  $\mathbf{pda} \geq \max(1, \mathbf{n})$ .
- 7: **b**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **b** must be at least  
 $\max(1, \mathbf{pdb} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{p} \times \mathbf{pdb})$  when **order** = Nag\_RowMajor.  
The ( $i, j$ )th element of the matrix  $B$  is stored in  
 $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$  when **order** = Nag\_RowMajor.  
*On entry:* the  $p$  by  $n$  matrix  $B$ .  
*On exit:* **b** is overwritten.
- 8: **pdb** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.  
*Constraints:*  
if **order** = Nag\_ColMajor,  $\mathbf{pdb} \geq \max(1, \mathbf{p})$ ;  
if **order** = Nag\_RowMajor,  $\mathbf{pdb} \geq \max(1, \mathbf{n})$ .
- 9: **c**[**m**] – double *Input/Output*  
*On entry:* the right-hand side vector  $c$  for the least squares part of the LSE problem.  
*On exit:* the residual sum of squares for the solution vector  $x$  is given by the sum of squares of elements  $\mathbf{c}[\mathbf{n} - \mathbf{p}]$ ,  $\mathbf{c}[\mathbf{n} - \mathbf{p} + 1]$ ,  $\dots$ ,  $\mathbf{c}[\mathbf{m} - 1]$ ; the remaining elements are overwritten.

- 10: **d[p]** – double *Input/Output*  
*On entry:* the right-hand side vector  $d$  for the equality constraints.  
*On exit:* **d** is overwritten.
- 11: **x[n]** – double *Output*  
*On exit:* the solution vector  $x$  of the LSE problem.
- 12: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.  
See Section 3.2.1.2 in the Essential Introduction for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **m** =  $\langle value \rangle$ .  
Constraint: **m**  $\geq$  0.

On entry, **n** =  $\langle value \rangle$ .  
Constraint: **n**  $\geq$  0.

On entry, **pda** =  $\langle value \rangle$ .  
Constraint: **pda**  $>$  0.

On entry, **pdb** =  $\langle value \rangle$ .  
Constraint: **pdb**  $>$  0.

### NE\_INT\_2

On entry, **pda** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .  
Constraint: **pda**  $\geq$   $\max(1, \mathbf{m})$ .

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
Constraint: **pda**  $\geq$   $\max(1, \mathbf{n})$ .

On entry, **pdb** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
Constraint: **pdb**  $\geq$   $\max(1, \mathbf{n})$ .

On entry, **pdb** =  $\langle value \rangle$  and **p** =  $\langle value \rangle$ .  
Constraint: **pdb**  $\geq$   $\max(1, \mathbf{p})$ .

### NE\_INT\_3

On entry, **p** =  $\langle value \rangle$ , **m** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
Constraint:  $0 \leq \mathbf{p} \leq \mathbf{n} \leq \mathbf{m} + \mathbf{p}$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
See Section 3.6.6 in the Essential Introduction for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 3.6.5 in the Essential Introduction for further information.

**NE\_SINGULAR**

The  $(N - P)$  by  $(N - P)$  part of the upper trapezoidal factor  $T$  associated with  $A$  in the generalized  $RQ$  factorization of the pair  $(B, A)$  is singular, so that the rank of the matrix  $(E)$  comprising the rows of  $A$  and  $B$  is less than  $n$ ; the least squares solutions could not be computed.

The upper triangular factor  $R$  associated with  $B$  in the generalized  $RQ$  factorization of the pair  $(B, A)$  is singular, so that  $\text{rank}(B) < p$ ; the least squares solution could not be computed.

**7 Accuracy**

For an error analysis, see Anderson *et al.* (1992) and Eldèn (1980). See also Section 4.6 of Anderson *et al.* (1999).

**8 Parallelism and Performance**

nag\_dgglse (f08zac) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dgglse (f08zac) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

When  $m \geq n = p$ , the total number of floating-point operations is approximately  $\frac{2}{3}n^2(6m + n)$ ; if  $p \ll n$ , the number reduces to approximately  $\frac{2}{3}n^2(3m - n)$ .

nag\_opt\_lin\_lsq (e04ncc) may also be used to solve LSE problems. It differs from nag\_dgglse (f08zac) in that it uses an iterative (rather than direct) method, and that it allows general upper and lower bounds to be specified for the variables  $x$  and the linear constraints  $Bx$ .

**10 Example**

This example solves the least squares problem

$$\underset{x}{\text{minimize}} \|c - Ax\|_2 \quad \text{subject to} \quad Bx = d$$

where

$$c = \begin{pmatrix} -1.50 \\ -2.14 \\ 1.23 \\ -0.54 \\ -1.68 \\ 0.82 \end{pmatrix},$$

$$A = \begin{pmatrix} -0.57 & -1.28 & -0.39 & 0.25 \\ -1.93 & 1.08 & -0.31 & -2.14 \\ 2.30 & 0.24 & 0.40 & -0.35 \\ -1.93 & 0.64 & -0.66 & 0.08 \\ 0.15 & 0.30 & 0.15 & -2.13 \\ -0.02 & 1.03 & -1.43 & 0.50 \end{pmatrix},$$

$$B = \begin{pmatrix} 1.0 & 0 & -1.0 & 0 \\ 0 & 1.0 & 0 & -1.0 \end{pmatrix}$$

and

$$d = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The constraints  $Bx = d$  correspond to  $x_1 = x_3$  and  $x_2 = x_4$ .

## 10.1 Program Text

```

/* nag_dgglse (f08zac) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagf16.h>

int main(void)
{
    /* Scalars */
    double      rnorm;
    Integer      i, j, m, n, p, pda, pdb;
    Integer      exit_status = 0;
    NagError     fail;
    Nag_OrderType order;
    /* Arrays */
    double      *a = 0, *b = 0, *c = 0, *d = 0, *x = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dgglse (f08zac) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%"NAG_IFMT%"NAG_IFMT%"NAG_IFMT"%*[\n] ", &m, &n, &p);

```

```

#else
    scanf("%NAG_IFMT%"NAG_IFMT%"NAG_IFMT"%*[\n] ", &m, &n, &p);
#endif

#ifdef NAG_COLUMN_MAJOR
    pda = m;
    pdb = p;
#else
    pda = n;
    pdb = n;
#endif

    /* Allocate memory */
    if (!(a = NAG_ALLOC(m*n, double)) ||
        !(b = NAG_ALLOC(p*n, double)) ||
        !(c = NAG_ALLOC(m, double)) ||
        !(d = NAG_ALLOC(p, double)) ||
        !(x = NAG_ALLOC(n, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A, B, C and D from data file */
    for (i = 1; i <= m; ++i)
    {
        for (j = 1; j <= n; ++j)
#ifdef _WIN32
            scanf_s("%lf", &A(i, j));
#else
            scanf("%lf", &A(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    for (i = 1; i <= p; ++i)
    {
        for (j = 1; j <= n; ++j)
#ifdef _WIN32
            scanf_s("%lf", &B(i, j));
#else
            scanf("%lf", &B(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    for (i = 1; i <= m; ++i)
#ifdef _WIN32
    scanf_s("%lf", &c[i - 1]);
#else
    scanf("%lf", &c[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    for (i = 1; i <= p; ++i)
#ifdef _WIN32
    scanf_s("%lf", &d[i - 1]);

```

```

#else
    scanf("%lf", &d[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Solve the equality-constrained least-squares problem */
/* minimize ||c - A*x|| (in the 2-norm) subject to B*x = D */
nag_dgglse(order, m, n, p, a, pda, b, pdb, c, d, x, &fail);

if (fail.code == NE_NOERROR)
{
    /* Print least-squares solution */
    printf("%s\n", "Constrained least-squares solution");
    for (i = 1; i <= n; ++i)
        printf("%11.4f%s", x[i - 1], i%7 == 0 || i == n?"\n":"" );

    /* Compute the square root of the residual sum of squares */
    nag_dge_norm(Nag_ColMajor, Nag_FrobeniusNorm, 1, m - n + p, &c[n - p], 1,
        &rnorm, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_dge_norm (f16rac).\n%s\n",
            fail.message);
        exit_status = 1;
        goto END;
    }
    printf("\nSquare root of the residual sum of squares\n");
    printf("%11.2e\n", rnorm);
}
else
{
    printf("Error from nag_dgglse (f08zac).\n%s\n", fail.message);
    exit_status = 1;
}

END:
NAG_FREE(a);
NAG_FREE(b);
NAG_FREE(c);
NAG_FREE(d);
NAG_FREE(x);

return exit_status;
}

```

## 10.2 Program Data

nag\_dgglse (f08zac) Example Program Data

```

6      4      2      :Values of M, N and P

-0.57 -1.28 -0.39  0.25
-1.93  1.08 -0.31 -2.14
 2.30  0.24  0.40 -0.35
-1.93  0.64 -0.66  0.08
 0.15  0.30  0.15 -2.13
-0.02  1.03 -1.43  0.50 :End of matrix A

 1.00  0.00 -1.00  0.00
 0.00  1.00  0.00 -1.00 :End of matrix B

-1.50
-2.14
 1.23
-0.54

```

```
-1.68
 0.82          :End of vector c

 0.00
 0.00          :End of vector d
```

### 10.3 Program Results

nag\_dgglse (f08zac) Example Program Results

```
Constrained least-squares solution
 0.4890      0.9975      0.4890      0.9975
```

```
Square root of the residual sum of squares
 2.51e-02
```

---