

# NAG Library Function Document

## nag\_ztrsna (f08qyc)

### 1 Purpose

nag\_ztrsna (f08qyc) estimates condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_ztrsna (Nag_OrderType order, Nag_JobType job,
  Nag_HowManyType how_many, const Nag_Boolean select[], Integer n,
  const Complex t[], Integer pdt, const Complex vl[], Integer pdvl,
  const Complex vr[], Integer pdvr, double s[], double sep[], Integer mm,
  Integer *m, NagError *fail)
```

### 3 Description

nag\_ztrsna (f08qyc) estimates condition numbers for specified eigenvalues and/or right eigenvectors of a complex upper triangular matrix  $T$ . These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix  $A = ZTZ^H$  (with unitary  $Z$ ), from which  $T$  may have been derived.

nag\_ztrsna (f08qyc) computes the reciprocal of the condition number of an eigenvalue  $\lambda_i$  as

$$s_i = \frac{|v^H u|}{\|u\|_E \|v\|_E},$$

where  $u$  and  $v$  are the right and left eigenvectors of  $T$ , respectively, corresponding to  $\lambda_i$ . This reciprocal condition number always lies between zero (i.e., ill-conditioned) and one (i.e., well-conditioned).

An approximate error estimate for a computed eigenvalue  $\lambda_i$  is then given by

$$\frac{\epsilon \|T\|}{s_i},$$

where  $\epsilon$  is the *machine precision*.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to  $\lambda_i$ , the function first calls nag\_ztrexc (f08qtc) to reorder the eigenvalues so that  $\lambda_i$  is in the leading position:

$$T = Q \begin{pmatrix} \lambda_i & c^H \\ 0 & T_{22} \end{pmatrix} Q^H.$$

The reciprocal condition number of the eigenvector is then estimated as  $sep_i$ , the smallest singular value of the matrix  $(T_{22} - \lambda_i I)$ . This number ranges from zero (i.e., ill-conditioned) to very large (i.e., well-conditioned).

An approximate error estimate for a computed right eigenvector  $u$  corresponding to  $\lambda_i$  is then given by

$$\frac{\epsilon \|T\|}{sep_i}.$$

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **job** – Nag\_JobType *Input*  
*On entry:* indicates whether condition numbers are required for eigenvalues and/or eigenvectors.  
**job** = Nag\_EigVals  
 Condition numbers for eigenvalues only are computed.  
**job** = Nag\_EigVecs  
 Condition numbers for eigenvectors only are computed.  
**job** = Nag\_DoBoth  
 Condition numbers for both eigenvalues and eigenvectors are computed.  
*Constraint:* **job** = Nag\_EigVals, Nag\_EigVecs or Nag\_DoBoth.
- 3: **how\_many** – Nag\_HowManyType *Input*  
*On entry:* indicates how many condition numbers are to be computed.  
**how\_many** = Nag\_ComputeAll  
 Condition numbers for all eigenpairs are computed.  
**how\_many** = Nag\_ComputeSelected  
 Condition numbers for selected eigenpairs (as specified by **select**) are computed.  
*Constraint:* **how\_many** = Nag\_ComputeAll or Nag\_ComputeSelected.
- 4: **select** $[dim]$  – const Nag\_Boolean *Input*  
**Note:** the dimension,  $dim$ , of the array **select** must be at least **n** when **how\_many** = Nag\_ComputeSelected; otherwise **select** may be **NULL**.  
*On entry:* specifies the eigenpairs for which condition numbers are to be computed if **how\_many** = Nag\_ComputeSelected. To select condition numbers for the eigenpair corresponding to the eigenvalue  $\lambda_j$ , **select** $[j - 1]$  must be set to Nag\_TRUE.  
 If **how\_many** = Nag\_ComputeAll, **select** is not referenced and may be **NULL**.
- 5: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $T$ .  
*Constraint:*  $n \geq 0$ .
- 6: **t** $[dim]$  – const Complex *Input*  
**Note:** the dimension,  $dim$ , of the array **t** must be at least **pd<sub>t</sub>**  $\times$  **n**.  
 The  $(i, j)$ th element of the matrix  $T$  is stored in  

$$\mathbf{t}[(j - 1) \times \mathbf{pd}_t + i - 1]$$
 when **order** = Nag\_ColMajor;  

$$\mathbf{t}[(i - 1) \times \mathbf{pd}_t + j - 1]$$
 when **order** = Nag\_RowMajor.  
*On entry:* the  $n$  by  $n$  upper triangular matrix  $T$ , as returned by nag\_zhseqr (f08psc).

- 7: **pdt** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **t**.  
*Constraint:* **pdt**  $\geq$  max(1, **n**).
- 8: **vl**[*dim*] – const Complex *Input*  
**Note:** the dimension, *dim*, of the array **vl** must be at least  
**pdvl**  $\times$  **mm** when **job** = Nag\_EigVals or Nag\_DoBoth and **order** = Nag\_ColMajor;  
**n**  $\times$  **pdvl** when **job** = Nag\_EigVals or Nag\_DoBoth and **order** = Nag\_RowMajor;  
otherwise **vl** may be **NULL**.  
The (*i*, *j*)th element of the matrix is stored in  
**vl**[(*j* – 1)  $\times$  **pdvl** + *i* – 1] when **order** = Nag\_ColMajor;  
**vl**[(*i* – 1)  $\times$  **pdvl** + *j* – 1] when **order** = Nag\_RowMajor.  
*On entry:* if **job** = Nag\_EigVals or Nag\_DoBoth, **vl** must contain the left eigenvectors of *T* (or of any matrix  $QTQ^H$  with *Q* unitary) corresponding to the eigenpairs specified by **how\_many** and **select**. The eigenvectors **must** be stored in consecutive rows or columns (depending on the value of **order**) of **vl**, as returned by nag\_zhsein (f08pxc) or nag\_ztrevc (f08qxc).  
If **job** = Nag\_EigVecs, **vl** is not referenced and may be **NULL**.
- 9: **pdvl** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **vl**.  
*Constraints:*  
if **order** = Nag\_ColMajor,  
if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvl**  $\geq$  **n**;  
if **job** = Nag\_EigVecs, **vl** may be **NULL**.;  
if **order** = Nag\_RowMajor,  
if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvl**  $\geq$  **mm**;  
if **job** = Nag\_EigVecs, **vl** may be **NULL**.
- 10: **vr**[*dim*] – const Complex *Input*  
**Note:** the dimension, *dim*, of the array **vr** must be at least  
**pdvr**  $\times$  **mm** when **job** = Nag\_EigVals or Nag\_DoBoth and **order** = Nag\_ColMajor;  
**n**  $\times$  **pdvr** when **job** = Nag\_EigVals or Nag\_DoBoth and **order** = Nag\_RowMajor;  
otherwise **vr** may be **NULL**.  
The (*i*, *j*)th element of the matrix is stored in  
**vr**[(*j* – 1)  $\times$  **pdvr** + *i* – 1] when **order** = Nag\_ColMajor;  
**vr**[(*i* – 1)  $\times$  **pdvr** + *j* – 1] when **order** = Nag\_RowMajor.  
*On entry:* if **job** = Nag\_EigVals or Nag\_DoBoth, **vr** must contain the right eigenvectors of *T* (or of any matrix  $QTQ^H$  with *Q* unitary) corresponding to the eigenpairs specified by **how\_many** and **select**. The eigenvectors **must** be stored in consecutive rows or columns (depending on the value of **order**) of **vr**, as returned by nag\_zhsein (f08pxc) or nag\_ztrevc (f08qxc).  
If **job** = Nag\_EigVecs, **vr** is not referenced and may be **NULL**.
- 11: **pdvr** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **vr**.

*Constraints:*

if **order** = Nag\_ColMajor,  
     if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvr**  $\geq$  **n**;  
     if **job** = Nag\_EigVecs, **vr** may be **NULL**.;  
 if **order** = Nag\_RowMajor,  
     if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvr**  $\geq$  **mm**;  
     if **job** = Nag\_EigVecs, **vr** may be **NULL**.

12: **s**[*dim*] – double *Output*

**Note:** the dimension, *dim*, of the array **s** must be at least

**mm** when **job** = Nag\_EigVals or Nag\_DoBoth;  
 otherwise **s** may be **NULL**.

*On exit:* the reciprocal condition numbers of the selected eigenvalues if **job** = Nag\_EigVals or Nag\_DoBoth, stored in consecutive elements of the array. Thus **s**[*j* – 1], **sep**[*j* – 1] and the *j*th rows or columns of **vl** and **vr** all correspond to the same eigenpair (but not in general the *j*th eigenpair unless all eigenpairs have been selected).

If **job** = Nag\_EigVecs, **s** is not referenced and may be **NULL**.

13: **sep**[*dim*] – double *Output*

**Note:** the dimension, *dim*, of the array **sep** must be at least

**mm** when **job** = Nag\_EigVecs or Nag\_DoBoth;  
 otherwise **sep** may be **NULL**.

*On exit:* the estimated reciprocal condition numbers of the selected right eigenvectors if **job** = Nag\_EigVecs or Nag\_DoBoth, stored in consecutive elements of the array.

If **job** = Nag\_EigVals, **sep** is not referenced and may be **NULL**.

14: **mm** – Integer *Input*

*On entry:* the number of elements in the arrays **s** and **sep**, and the number of rows or columns (depending on the value of **order**) in the arrays **vl** and **vr** (if used). The precise number required, *required<sub>r,owcol</sub>*, is *n* if **how\_many** = Nag\_ComputeAll; if **how\_many** = Nag\_ComputeSelected, *required<sub>r,owcol</sub>* is the number of selected eigenpairs (see **select**), in which case  $0 \leq \text{required}_{r,owcol} \leq n$ .

*Constraints:*

if **how\_many** = Nag\_ComputeAll, **mm**  $\geq$  **n**;  
 otherwise **mm**  $\geq$  *required<sub>r,owcol</sub>*.

15: **m** – Integer \* *Output*

*On exit:* *required<sub>r,owcol</sub>*, the number of selected eigenpairs. If **how\_many** = Nag\_ComputeAll, **m** is set to *n*.

16: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_ENUM\_INT\_2**

On entry, **how\_many** =  $\langle value \rangle$ , **mm** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: if **how\_many** = Nag\_ComputeAll, **mm**  $\geq$  **n**;  
 otherwise **mm**  $\geq$  *required\_rowcol*.

On entry, **job** =  $\langle value \rangle$ , **pdvl** =  $\langle value \rangle$ , **mm** =  $\langle value \rangle$ .  
 Constraint: if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvl**  $\geq$  **mm**.

On entry, **job** =  $\langle value \rangle$ , **pdvl** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvl**  $\geq$  **n**.

On entry, **job** =  $\langle value \rangle$ , **pdvr** =  $\langle value \rangle$ , **mm** =  $\langle value \rangle$ .  
 Constraint: if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvr**  $\geq$  **mm**.

On entry, **job** =  $\langle value \rangle$ , **pdvr** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: if **job** = Nag\_EigVals or Nag\_DoBoth, **pdvr**  $\geq$  **n**.

**NE\_INT**

On entry, **n** =  $\langle value \rangle$ .  
 Constraint: **n**  $\geq$  0.

On entry, **pdv** =  $\langle value \rangle$ .  
 Constraint: **pdv**  $>$  0.

On entry, **pdvl** =  $\langle value \rangle$ .  
 Constraint: **pdvl**  $>$  0.

On entry, **pdvr** =  $\langle value \rangle$ .  
 Constraint: **pdvr**  $>$  0.

**NE\_INT\_2**

On entry, **pdv** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pdv**  $\geq$   $\max(1, \mathbf{n})$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 3.6.6 in the Essential Introduction for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
 See Section 3.6.5 in the Essential Introduction for further information.

**7 Accuracy**

The computed values  $sep_i$  may over estimate the true value, but seldom by a factor of more than 3.

**8 Parallelism and Performance**

nag\_ztrsna (f08qyc) is not threaded by NAG in any implementation.

nag\_ztrsna (f08qyc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The real analogue of this function is nag\_dtrsna (f08qlc).

## 10 Example

This example computes approximate error estimates for all the eigenvalues and right eigenvectors of the matrix  $T$ , where

$$T = \begin{pmatrix} -6.0004 - 6.9999i & 0.3637 - 0.3656i & -0.1880 + 0.4787i & 0.8785 - 0.2539i \\ 0.0000 + 0.0000i & -5.0000 + 2.0060i & -0.0307 - 0.7217i & -0.2290 + 0.1313i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 7.9982 - 0.9964i & 0.9357 + 0.5359i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 3.0023 - 3.9998i \end{pmatrix}.$$

### 10.1 Program Text

```

/* nag_ztrsna (f08qyc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf08.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    Integer      i, j, m, n, pdt, pdvl, pdvr;
    Integer      s_len;
    Integer      exit_status = 0;
    double       eps, tnorm;
    NagError     fail;
    Nag_OrderType order;
    /* Arrays */
    double       *s = 0, *sep = 0;
    Complex      *t = 0, *vl = 0, *vr = 0;

#ifdef NAG_COLUMN_MAJOR
#define T(I, J) t[(J-1)*pdt + I - 1]
    order = Nag_ColMajor;
#else
#define T(I, J) t[(I-1)*pdt + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_ztrsna (f08qyc) Example Program Results\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}

```

```

    scanf_s("%NAG_IFMT"%*[^\\n] ", &n);
#else
    scanf("%NAG_IFMT"%*[^\\n] ", &n);
#endif
#ifdef NAG_COLUMN_MAJOR
    pdt = n;
    pdvl = n;
    pdvr = n;
#else
    pdt = n;
    pdvl = n;
    pdvr = n;
#endif
    s_len = n;

    /* Allocate memory */
    if (!(t = NAG_ALLOC(n * n, Complex)) ||
        !(vl = NAG_ALLOC(n * n, Complex)) ||
        !(vr = NAG_ALLOC(n * n, Complex)) ||
        !(s = NAG_ALLOC(s_len, double)) ||
        !(sep = NAG_ALLOC(s_len, double)))
    {
        printf("Allocation failure\\n");
        exit_status = -1;
        goto END;
    }

    /* Read T from data file */
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= n; ++j)
#ifdef _WIN32
            scanf_s(" ( %lf , %lf ) ", &T(i, j).re, &T(i, j).im);
#else
            scanf(" ( %lf , %lf ) ", &T(i, j).re, &T(i, j).im);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[^\\n] ");
#else
    scanf("%*[^\\n] ");
#endif

    /* Calculate right and left eigenvectors of T */
    /* nag_ztrevc (f08qxc).
    * Left and right eigenvectors of complex upper triangular
    * matrix
    */
    nag_ztrevc(order, Nag_BothSides, Nag_ComputeAll, NULL, n, t, pdt,
              vl, pdvl, vr, pdvr, n, &m, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_ztrevc (f08qxc).\\n%s\\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Estimate condition numbers for all the eigenvalues and */
    /* right eigenvectors of T */
    /* nag_ztrsna (f08qyc).
    * Estimates of sensitivities of selected eigenvalues and
    * eigenvectors of complex upper triangular matrix
    */
    nag_ztrsna(order, Nag_DoBoth, Nag_ComputeAll, NULL, n, t, pdt,
              vl, pdvl, vr, pdvr, s, sep, n, &m, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_ztrsna (f08qyc).\\n%s\\n", fail.message);
        exit_status = 1;
        goto END;
    }
}

```

```

/* Print condition numbers of eigenvalues and right eigenvectors */
printf("\nS\n");
for (i = 0; i < n; ++i)
    printf("%11.1e", s[i]);
printf("\n\nSep\n");
for (i = 0; i < n; ++i)
    printf("%11.1e", sep[i]);
printf("\n");
/* Calculate approximate error estimates (using the 1-norm) */
/* nag_zge_norm (f16uac).
 * 1-norm, infinity-norm, Frobenius norm, largest absolute
 * element, complex general matrix
 */
nag_zge_norm(order, Nag_OneNorm, n, n, t, pdt, &tnorm, &fail);
if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_zge_norm (f16uac).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }
/* nag_machine_precision (x02ajc).
 * The machine precision
 */
eps = nag_machine_precision;
printf("\nApproximate error estimates for eigenvalues"
       "of T (machine dependent)\n");
for (i = 0; i < m; ++i)
    printf("%11.1e", eps*tnorm/s[i]);
printf("\n\nApproximate error estimates for right eigenvectors"
       "of T (machine dependent)\n");
for (i = 0; i < m; ++i)
    printf("%11.1e", eps*tnorm/sep[i]);
printf("\n");
END:
NAG_FREE(t);
NAG_FREE(s);
NAG_FREE(sep);
NAG_FREE(vl);
NAG_FREE(vr);

return exit_status;
}

```

## 10.2 Program Data

```

nag_ztrsna (f08qyc) Example Program Data
4
(-6.0004,-6.9999) ( 0.3637,-0.3656) (-0.1880, 0.4787) ( 0.8785,-0.2539)
( 0.0000, 0.0000) (-5.0000, 2.0060) (-0.0307,-0.7217) (-0.2290, 0.1313)
( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 7.9982,-0.9964) ( 0.9357, 0.5359)
( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 0.0000, 0.0000) ( 3.0023,-3.9998)
:Value of N
:End of matrix T

```

## 10.3 Program Results

```

nag_ztrsna (f08qyc) Example Program Results

S
  9.9e-01   1.0e+00   9.8e-01   9.8e-01

Sep
  8.4e+00   8.0e+00   5.8e+00   5.8e+00

```



Approximate error estimates for eigenvalues of T (machine dependent)

1.0e-15    1.0e-15    1.1e-15    1.1e-15

Approximate error estimates for right eigenvectors of T (machine dependent)

1.2e-16    1.3e-16    1.8e-16    1.8e-16

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