NAG Library Function Document

nag_zpftrs (f07wsc)

1 Purpose

nag_zpftrs (f07wsc) solves a complex Hermitian positive definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

using the Cholesky factorization computed by nag_zpftrf (f07wrc) stored in Rectangular Full Packed (RFP) format.

2 Specification

```
#include <nag.h>
#include <nagf07.h>
void nag_zpftrs (Nag_OrderType order, Nag_RFP_Store transr,
     Nag_UploType uplo, Integer n, Integer nrhs, const Complex ar[],
     Complex b[], Integer pdb, NagError *fail)
```

3 Description

nag_zpftrs (f07wsc) is used to solve a complex Hermitian positive definite system of linear equations $AX = B$, the function must be preceded by a call to nag_zpftrf (f07wrc) which computes the Cholesky factorization of A, stored in RFP format. The RFP storage format is described in Section 3.3.3 in the f07 Chapter Introduction. The solution X is computed by forward and backward substitution.

If **uplo** = Nag_Upper, $A = U^{\mathrm{H}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{H}}Y = B$ and then $UX = Y$.

If **uplo** = Nag_Lower, $A = LL^{\mathrm{H}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{H}}X = Y$.

4 References

Gustavson F G, Waśniewski J, Dongarra J J and Langou J (2010) Rectangular full packed format for Cholesky's algorithm: factorization, solution, and inversion *ACM Trans. Math. Software* **37, 2**

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **transr** – Nag_RFP_Store *Input*

On entry: specifies whether the normal RFP representation of A or its conjugate transpose is stored.

transr = Nag_RFP_Normal
The matrix A is stored in normal RFP format.

transr = Nag_RFP_ConjTrans
The conjugate transpose of the RFP representation of the matrix A is stored.

Constraint: **transr** = Nag_RFP_Normal or Nag_RFP_ConjTrans.

3: **uplo** – Nag_UploType *Input*

On entry: specifies how A has been factorized.

uplo = Nag_Upper
$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = Nag_Lower
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = Nag_Upper or Nag_Lower.

4: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: $\mathbf{nrhs} \geq 0$.

6: **ar**$[\mathbf{n} \times (\mathbf{n}+\mathbf{1})/\mathbf{2}]$ – const Complex *Input*

On entry: the Cholesky factorization of A stored in RFP format, as returned by nag_zpftrf (f07wrc).

7: **b**$[dim]$ – Complex *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least

$\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{n} \times \mathbf{pdb})$ when **order** = Nag_RowMajor.

The (i,j)th element of the matrix B is stored in

b$[(j-1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
b$[(i-1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = Nag_ColMajor, $\mathbf{pdb} \geq \max(1, \mathbf{n})$;
if **order** = Nag_RowMajor, $\mathbf{pdb} \geq \max(1, \mathbf{nrhs})$.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

NE_INT_2

On entry, **pdb** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩ and **nrhs** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = Nag_Upper, $|E| \leq c(n)\epsilon|U^{\mathrm{H}}||U|$;

if **uplo** = Nag_Lower, $|E| \leq c(n)\epsilon|L||L^{\mathrm{H}}|$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$ and $\kappa_\infty(A)$ is the condition number when using the ∞-norm.

Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$.

8 Parallelism and Performance

nag_zpftrs (f07wsc) is not threaded by NAG in any implementation.

nag_zpftrs (f07wsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

The real analogue of this function is nag_dpftrs (f07wec).

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3.93 - 6.14i & 1.48 + 6.58i \\ 6.17 + 9.42i & 4.65 - 4.75i \\ -7.17 - 21.83i & -4.91 + 2.29i \\ 1.99 - 14.38i & 7.64 - 10.79i \end{pmatrix}.$$

Here A is Hermitian positive definite, stored in RFP format, and must first be factorized by nag_zpftrf (f07wrc).

10.1 Program Text

```
/* nag_zpftrs (f07wsc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 25, 2014.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer        exit_status = 0;
  Integer        i, j, k, lar1, lar2, lenar, n, nrhs, pdar, pdb, q;
  /* Arrays */
  Complex        *ar = 0, *b = 0;
  char           nag_enum_arg[40];
  /* NAG types */
  Nag_RFP_Store  transr;
  Nag_UploType   uplo;
  Nag_OrderType  order;
  NagError       fail;
  Nag_MatrixType matrix = Nag_GeneralMatrix;
  Nag_DiagType   diag = Nag_NonUnitDiag;

#ifdef NAG_COLUMN_MAJOR
  order = Nag_ColMajor;
#define AR(I,J) ar[J*pdar + I]
```

```
#define B(I, J) b[J*pdb + I]
#else
  order = Nag_RowMajor;
#define AR(I,J) ar[I*pdar + J]
#define B(I, J) b[I*pdb + J]
#endif
  INIT_FAIL(fail);

  printf("nag_zpftrs (f07wsc) Example Program Results\n\n");
  /* Skip heading in data file*/
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%"NAG_IFMT"", &n, &nrhs);
#else
  scanf("%"NAG_IFMT"%"NAG_IFMT"", &n, &nrhs);
#endif
#ifdef _WIN32
  scanf_s("%39s", nag_enum_arg, _countof(nag_enum_arg));
#else
  scanf("%39s", nag_enum_arg);
#endif
  uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
  scanf_s("%39s%*[^\n]", nag_enum_arg, _countof(nag_enum_arg));
#else
  scanf("%39s%*[^\n]", nag_enum_arg);
#endif
  transr = (Nag_RFP_Store) nag_enum_name_to_value(nag_enum_arg);

  lenar = (n * (n + 1))/2;
  if (!(ar = NAG_ALLOC(lenar, Complex)) ||
      !(b = NAG_ALLOC(n*nrhs, Complex)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Setup dimensions for RFP array ar and aray b. */
  k = n/2;
  q = n - k;
  if (transr==Nag_RFP_Normal) {
    lar1 = 2*k+1;
    lar2 = q;
  } else {
    lar1 = q;
    lar2 = 2*k+1;
  }
  if (order==Nag_RowMajor) {
    pdar = lar2;
    pdb = nrhs;
  } else {
    pdar = lar1;
    pdb = n;
  }
  /* Read matrix into RFP array ar. */
  for (i = 0; i < lar1; i++) {
    for (j = 0; j < lar2; j++) {
#ifdef _WIN32
      scanf_s(" ( %lf , %lf ) ", &AR(i,j).re, &AR(i,j).im);
#else
      scanf(" ( %lf , %lf ) ", &AR(i,j).re, &AR(i,j).im);
#endif
    }
  }
#ifdef _WIN32
  scanf_s("%*[^\n] ");
```

```
#else
  scanf("%*[^\n] ");
#endif

  /* Read B */
  for (i = 0; i < n; i++)
    for (j = 0; j < nrhs; j++)
#ifdef _WIN32
      scanf_s(" ( %lf , %lf ) ", &B(i, j).re, &B(i, j).im);
#else
      scanf(" ( %lf , %lf ) ", &B(i, j).re, &B(i, j).im);
#endif

  /* Factorize A using nag_zpftrf (f07wrc) which performs a Cholesky
   * factorization of a complex Hermitian positive definite matrix in
   * Rectangular Full Packed format.
   */
  nag_zpftrf(order, transr, uplo, n, ar, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_zpftrf (f07wrc)\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* Compute solution Ax = B using nag_zpftrs (f07wsc). */
  nag_zpftrs(order, transr, uplo, n, nrhs, ar, b, pdb, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_zpftrs (f07wsc)\n%s\n", fail.message);
    exit_status = 2;
    goto END;
  }

  /* nag_gen_complx_mat_print_comp (x04dbc).
   * Print complex general matrix (comprehensive).
   */
  nag_gen_complx_mat_print_comp(order, matrix, diag, n, nrhs, b, pdb,
                                Nag_BracketForm, "%7.4f", "Solution(s)",
                                Nag_IntegerLabels, NULL,
                                Nag_IntegerLabels, NULL, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc)\n%s\n",
           fail.message);
    exit_status = 3;
  }

 END:
  NAG_FREE(ar);
  NAG_FREE(b);
  return exit_status;
}
```

10.2 Program Data

```
nag_zpftrs (f07wsc) Example Program Data
   4      2
   Nag_Lower
   Nag_RFP_Normal                 : n, nrhs, uplo, transr

 ( 4.09,  0.00) ( 2.33, -0.14)
 ( 3.23,  0.00) ( 4.29,  0.00)
 ( 1.51,  1.92) ( 3.58,  0.00)
 ( 1.90, -0.84) (-0.23, -1.11)
 ( 0.42, -2.50) (-1.18, -1.37)     : ar[]

 ( 3.93, -6.14) ( 1.48,  6.58)
 ( 6.17,  9.42) ( 4.65, -4.75)
 (-7.17,-21.83) (-4.91,  2.29)
 ( 1.99,-14.38) ( 7.64,-10.79)     : B
```

10.3 Program Results

```
nag_zpftrs (f07wsc) Example Program Results

 Solution(s)
                     1                   2
 1  ( 1.0000,-1.0000)  (-1.0000, 2.0000)
 2  (-0.0000, 3.0000)  ( 3.0000,-4.0000)
 3  (-4.0000,-5.0000)  (-2.0000, 3.0000)
 4  ( 2.0000, 1.0000)  ( 4.0000,-5.0000)
```