NAG Library Function Document

nag_zposv (f07fnc)

1 Purpose

nag_zposv (f07fnc) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive definite matrix and X and B are n by r matrices.

2 Specification

```
#include <nag.h>
#include <nagf07.h>
void nag_zposv (Nag_OrderType order, Nag_UploType uplo, Integer n,
     Integer nrhs, Complex a[], Integer pda, Complex b[], Integer pdb,
     NagError *fail)
```

3 Description

nag_zposv (f07fnc) uses the Cholesky decomposition to factor A as $A = U^{\mathrm{H}}U$ if **uplo** = Nag_Upper or $A = LL^{\mathrm{H}}$ if **uplo** = Nag_Lower, where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **uplo** – Nag_UploType *Input*

On entry: if **uplo** = Nag_Upper, the upper triangle of A is stored.

If **uplo** = Nag_Lower, the lower triangle of A is stored.

Constraint: **uplo** = Nag_Upper or Nag_Lower.

3: **n** – Integer *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: **n** ≥ 0.

4: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: **nrhs** ≥ 0.

5: **a**$[dim]$ – Complex *Input/Output*

Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \textbf{pda} \times \textbf{n})$.

On entry: the n by n Hermitian matrix A.

If **order** = Nag_ColMajor, A_{ij} is stored in **a**$[(j-1) \times \textbf{pda} + i - 1]$.

If **order** = Nag_RowMajor, A_{ij} is stored in **a**$[(i-1) \times \textbf{pda} + j - 1]$.

If **uplo** = Nag_Upper, the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = Nag_Lower, the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

On exit: if **fail.code** = NE_NOERROR, the factor U or L from the Cholesky factorization $A = U^{\mathrm{H}}U$ or $A = LL^{\mathrm{H}}$.

6: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **a**.

Constraint: **pda** $\geq \max(1, \textbf{n})$.

7: **b**$[dim]$ – Complex *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least

$\max(1, \textbf{pdb} \times \textbf{nrhs})$ when **order** = Nag_ColMajor;
$\max(1, \textbf{n} \times \textbf{pdb})$ when **order** = Nag_RowMajor.

The (i,j)th element of the matrix B is stored in

b$[(j-1) \times \textbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
b$[(i-1) \times \textbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the n by r right-hand side matrix B.

On exit: if **fail.code** = NE_NOERROR, the n by r solution matrix X.

8: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = Nag_ColMajor, **pdb** $\geq \max(1, \textbf{n})$;
if **order** = Nag_RowMajor, **pdb** $\geq \max(1, \textbf{nrhs})$.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pda** = ⟨*value*⟩.
Constraint: **pda** > 0.

On entry, **pdb** = ⟨*value*⟩.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pda** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩ and **nrhs** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_MAT_NOT_POS_DEF

The leading minor of order ⟨*value*⟩ of A is not positive definite, so the factorization could not be completed, and the solution has not been computed.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

nag_zposvx (f07fpc) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag_herm_posdef_lin_solve (f04cdc)

solves $Ax = b$ and returns a forward error bound and condition estimate. nag_herm_posdef_lin_solve (f04cdc) calls nag_zposv (f07fnc) to solve the equations.

8 Parallelism and Performance

nag_zposv (f07fnc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_zposv (f07fnc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this function is nag_dposv (f07fac).

10 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric positive definite matrix

$$A = \begin{pmatrix} 3.23 & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 3.93 - 6.14i \\ 6.17 + 9.42i \\ -7.17 - 21.83i \\ 1.99 - 14.38i \end{pmatrix}.$$

Details of the Cholesky factorization of A are also output.

10.1 Program Text

```
/* nag_zposv (f07fnc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer       exit_status = 0, i, j, n, nrhs, pda, pdb;
```

```
  /* Arrays */
  Complex       *a = 0, *b = 0;

  /* Nag Types */
  NagError      fail;
  Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_zposv (f07fnc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%"NAG_IFMT"%*[^\n]", &n, &nrhs);
#else
  scanf("%"NAG_IFMT"%"NAG_IFMT"%*[^\n]", &n, &nrhs);
#endif
  if (n < 0 || nrhs < 0)
    {
      printf("Invalid n or nrhs\n");
      exit_status = 1;
      goto END;
    }

  pda = n;
#ifdef NAG_COLUMN_MAJOR
  pdb = n;
#else
  pdb = nrhs;
#endif

  /* Allocate memory */
  if (!(a = NAG_ALLOC(n * n, Complex)) ||
      !(b = NAG_ALLOC(n*nrhs, Complex)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read the upper triangular part of A from data file */

  for (i = 1; i <= n; ++i)
    for (j = i; j <= n; ++j)
#ifdef _WIN32
      scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
      scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
```

```
  /* Read b from data file */
  for (i = 1; i <= n; ++i)
    for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
      scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
      scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  /* Solve the equations Ax = b for x nag_zposv (f07fnc). */
  nag_zposv(order, Nag_Upper, n, nrhs, a, pda, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_zposv (f07fnc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print solution */
  printf("Solution\n");
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        printf("(%7.4f, %7.4f)%s", B(i, j).re, B(i, j).im, j%4 == 3?"\n":" ");
      printf("\n");
    }

  /* Print details of factorization using
   * nag_gen_complx_mat_print_comp (x04dbc).
   */
  printf("\n");
  fflush(stdout);
  nag_gen_complx_mat_print_comp(order, Nag_UpperMatrix, Nag_NonUnitDiag, n, n,
                                a, pda, Nag_BracketForm, "%7.4f",
                                "Cholesky factor U", Nag_IntegerLabels, 0,
                                Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }

 END:
  NAG_FREE(a);
  NAG_FREE(b);

  return exit_status;
}

#undef A
#undef B
```

10.2 Program Data

```
nag_zposv (f07fnc) Example Program Data
   4           1                                          : n, nrhs
 ( 3.23,  0.00) ( 1.51, -1.92) ( 1.90,  0.84) ( 0.42,  2.50)
                ( 3.58,  0.00) (-0.23,  1.11) (-1.18,  1.37)
                               ( 4.09,  0.00) ( 2.33, -0.14)
                                              ( 4.29,  0.00) : matrix A
 ( 3.93, -6.14) ( 6.17,  9.42) (-7.17,-21.83) ( 1.99,-14.38) : vector b
```

10.3 Program Results

```
nag_zposv (f07fnc) Example Program Results

Solution
( 1.0000, -1.0000)
(-0.0000,  3.0000)
(-4.0000, -5.0000)
( 2.0000,  1.0000)

 Cholesky factor U
                    1                    2                    3                    4
 1  ( 1.7972, 0.0000)  ( 0.8402,-1.0683)  ( 1.0572, 0.4674)  ( 0.2337, 1.3910)
 2                     ( 1.3164, 0.0000)  (-0.4702,-0.3131)  ( 0.0834,-0.0368)
 3                                        ( 1.5604, 0.0000)  ( 0.9360,-0.9900)
 4                                                           ( 0.6603, 0.0000)
```