

NAG Library Function Document

nag_dgbcon (f07bgc)

1 Purpose

nag_dgbcon (f07bgc) estimates the condition number of a real band matrix A , where A has been factorized by nag_dgbtrf (f07bdc).

2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dgbcon (Nag_OrderType order, Nag_NormType norm, Integer n,
                Integer kl, Integer ku, const double ab[], Integer pdab,
                const Integer ipiv[], double anorm, double *rcond, NagError *fail)
```

3 Description

nag_dgbcon (f07bgc) estimates the condition number of a real band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function should be preceded by a call to nag_dgb_norm (f16rbc) to compute $\|A\|_1$ or $\|A\|_\infty$, and a call to nag_dgbtrf (f07bdc) to compute the LU factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Arguments

- 1: **order** – Nag_OrderType *Input*
On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.
Constraint: **order** = Nag_RowMajor or Nag_ColMajor.
- 2: **norm** – Nag_NormType *Input*
On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.
norm = Nag_OneNorm
 $\kappa_1(A)$ is estimated.

- norm** = Nag_InfNorm
 $\kappa_{\infty}(A)$ is estimated.
Constraint: **norm** = Nag_OneNorm or Nag_InfNorm.
- 3: **n** – Integer *Input*
On entry: n , the order of the matrix A .
Constraint: $n \geq 0$.
- 4: **kl** – Integer *Input*
On entry: k_l , the number of subdiagonals within the band of the matrix A .
Constraint: $kl \geq 0$.
- 5: **ku** – Integer *Input*
On entry: k_u , the number of superdiagonals within the band of the matrix A .
Constraint: $ku \geq 0$.
- 6: **ab**[*dim*] – const double *Input*
Note: the dimension, *dim*, of the array **ab** must be at least $\max(1, \mathbf{pdab} \times \mathbf{n})$.
On entry: the LU factorization of A , as returned by nag_dgbtrf (f07bdc).
- 7: **pdab** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **ab**.
Constraint: $\mathbf{pdab} \geq 2 \times \mathbf{kl} + \mathbf{ku} + 1$.
- 8: **ipiv**[*dim*] – const Integer *Input*
Note: the dimension, *dim*, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.
On entry: the pivot indices, as returned by nag_dgbtrf (f07bdc).
- 9: **anorm** – double *Input*
On entry: if **norm** = Nag_OneNorm, the 1-norm of the **original** matrix A .
 If **norm** = Nag_InfNorm, the ∞ -norm of the **original** matrix A .
anorm may be computed by calling nag_dgb_norm (f16rbc) with the same value for the argument **norm**.
anorm must be computed either **before** calling nag_dgbtrf (f07bdc) or else from a **copy** of the original matrix A (see Section 10).
Constraint: **anorm** ≥ 0.0 .
- 10: **rcond** – double * *Output*
On exit: an estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.
- 11: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{kl} = \langle value \rangle$.
Constraint: $\mathbf{kl} \geq 0$.

On entry, $\mathbf{ku} = \langle value \rangle$.
Constraint: $\mathbf{ku} \geq 0$.

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} \geq 0$.

On entry, $\mathbf{pdab} = \langle value \rangle$.
Constraint: $\mathbf{pdab} > 0$.

NE_INT_3

On entry, $\mathbf{pdab} = \langle value \rangle$, $\mathbf{kl} = \langle value \rangle$ and $\mathbf{ku} = \langle value \rangle$.
Constraint: $\mathbf{pdab} \geq 2 \times \mathbf{kl} + \mathbf{ku} + 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL

On entry, $\mathbf{anorm} = \langle value \rangle$.
Constraint: $\mathbf{anorm} \geq 0.0$.

7 Accuracy

The computed estimate \mathbf{rcond} is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where \mathbf{rcond} is much larger.

8 Parallelism and Performance

nag_dgbcon (f07bgc) is not threaded by NAG in any implementation.

nag_dgbcon (f07bgc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

A call to `nag_dgbcon` (f07bgc) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n(2k_l + k_u)$ floating-point operations (assuming $n \gg k_l$ and $n \gg k_u$) but takes considerably longer than a call to `nag_dgbtrs` (f07bec) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is `nag_zgbcon` (f07buc).

10 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by `nag_dgbtrf` (f07bdc). The true condition number in the 1-norm is 56.40.

10.1 Program Text

```

/* nag_dgbcon (f07bgc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    Integer    i, ipiv_len, j, kl, ku, n, pdab;
    Integer    exit_status = 0;
    double     anorm, rcond, sum;
    NagError   fail;
    Nag_OrderType order;

    /* Arrays */
    double     *ab = 0;
    Integer     *ipiv = 0;

#ifdef NAG_COLUMN_MAJOR
#define AB(I, J) ab[(J-1)*pdab + kl + ku + I - J]
    order = Nag_ColMajor;
#else
#define AB(I, J) ab[(I-1)*pdab + kl + J - I]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dgbcon (f07bgc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");

```

```

#endif
#ifdef _WIN32
    scanf_s("%"NAG_IFMT%"NAG_IFMT%"NAG_IFMT"%*[\n] ", &n, &kl, &ku);
#else
    scanf("%"NAG_IFMT%"NAG_IFMT%"NAG_IFMT"%*[\n] ", &n, &kl, &ku);
#endif
ipiv_len = n;
pdab = 2*kl + ku + 1;

/* Allocate memory */
if (!(ab = NAG_ALLOC((2*kl+ku+1) * n, double)) ||
    !(ipiv = NAG_ALLOC(ipiv_len, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read AB from data file */
for (i = 1; i <= n; ++i)
{
    for (j = MAX(i-kl, 1); j <= MIN(i+ku, n); ++j)
#ifdef _WIN32
        scanf_s("%lf", &AB(i, j));
#else
        scanf("%lf", &AB(i, j));
#endif
}
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
/* Compute norm of A */
anorm = 0.0;
for (j = 1; j <= n; ++j)
{
    sum = 0.0;
    for (i = MAX(j-ku, 1); i <= MIN(j+kl, n); ++i)
        sum = sum + ABS(AB(i, j));
    anorm = MAX(anorm, sum);
}
/* Factorize A */
/* nag_dgbtrf (f07bdc).
 * LU factorization of real m by n band matrix
 */
nag_dgbtrf(order, n, n, kl, ku, ab, pdab, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dgbtrf (f07bdc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Estimate condition number */
/* nag_dgbcon (f07bgc).
 * Estimate condition number of real band matrix, matrix
 * already factorized by nag_dgbtrf (f07bdc)
 */
nag_dgbcon(order, Nag_OneNorm, n, kl, ku, ab, pdab, ipiv,
           anorm, &rcond, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dgbcon (f07bgc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Print condition number */
/* nag_machine_precision (x02ajc).
 * The machine precision
 */
if (rcond > nag_machine_precision)

```

```
    printf("Estimate of condition number = %11.2e\n", 1.0/rcond);
else
    printf("A is singular to working precision\n");
END:
NAG_FREE(ab);
NAG_FREE(ipiv);
return exit_status;
}
```

10.2 Program Data

```
nag_dgbcon (f07bgc) Example Program Data
  4  1  2                               :Values of N, KL and KU
-0.23  2.54 -3.66
-6.98  2.46 -2.73 -2.13
        2.56  2.46  4.07
        -4.78 -3.82   :End of matrix A
```

10.3 Program Results

```
nag_dgbcon (f07bgc) Example Program Results
```

```
Estimate of condition number =    5.64e+01
```
