NAG Library Function Document

nag_1d_pade_eval (e02rbc)

1 Purpose

nag_1d_pade_eval (e02rbc) evaluates a rational function at a user-supplied point, given the numerator and denominator coefficients.

2 Specification

```
#include <nag.h>
#include <nage02.h>
void nag_1d_pade_eval (const double a[], Integer ia, const double b[],
     Integer ib, double x, double *ans, NagError *fail)
```

3 Description

Given a real value x and the coefficients a_j, for $j = 0, 1, \ldots, l$ and b_k, for $k = 0, 1, \ldots, m$, nag_1d_pade_eval (e02rbc) evaluates the rational function

$$\frac{\sum_{j=0}^{l} a_j x^j}{\sum_{k=0}^{m} b_k x^k}.$$

using nested multiplication (see Conte and de Boor (1965)).

A particular use of nag_1d_pade_eval (e02rbc) is to compute values of the Padé approximants determined by nag_1d_pade (e02rac).

4 References

Conte S D and de Boor C (1965) *Elementary Numerical Analysis* McGraw–Hill

Peters G and Wilkinson J H (1971) Practical problems arising in the solution of polynomial equations *J. Inst. Maths. Applics.* **8** 16–35

5 Arguments

1: **a**[**ia**] – const double *Input*

On entry: **a**[j], for $j = 1, 2, \ldots, l+1$, must contain the value of the coefficient a_j in the numerator of the rational function.

2: **ia** – Integer *Input*

On entry: the value of $l + 1$, where l is the degree of the numerator.

Constraint: **ia** ≥ 1.

3: **b**[**ib**] – const double *Input*

On entry: **b**[k], for $k = 1, 2, \ldots, m+1$, must contain the value of the coefficient b_k in the denominator of the rational function.

Constraint: if **ib** $= 1$, **b**[0] $\neq 0.0$.

4: **ib** – Integer *Input*

On entry: the value of $m+1$, where m is the degree of the denominator.

Constraint: **ib** ≥ 1.

5: **x** – double *Input*

On entry: the point x at which the rational function is to be evaluated.

6: **ans** – double * *Output*

On exit: the result of evaluating the rational function at the given point x.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **ia** = ⟨*value*⟩.
Constraint: **ia** ≥ 1.

On entry, **ib** = ⟨*value*⟩.
Constraint: **ib** ≥ 1.

NE_INT_ARRAY

The first **ib** entries in **b** are zero: **ib** = ⟨*value*⟩.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_POLE_PRESENT

Evaluation at or near a pole.

7 Accuracy

A running error analysis for polynomial evaluation by nested multiplication using the recurrence suggested by Kahan (see Peters and Wilkinson (1971)) is used to detect whether you are attempting to evaluate the approximant at or near a pole.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken is approximately proportional to $l + m$.

10 Example

This example first calls nag_1d_pade (e02rac) to calculate the $4/4$ Padé approximant to e^x, and then uses nag_1d_pade_eval (e02rbc) to evaluate the approximant at $x = 0.1, 0.2, \ldots, 1.0$.

10.1 Program Text

```
/* nag_1d_pade_eval (e02rbc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nage02.h>

int main(void)
{
  /* Scalars */
  double   ans, tval, x;
  Integer  exit_status, i, l, m, ia, ib, ic;
  NagError fail;

  /* Arrays */
  double   *aa = 0, *bb = 0, *cc = 0;

  INIT_FAIL(fail);

  exit_status = 0;
  printf("nag_1d_pade_eval (e02rbc) Example Program Results\n");

  l = 4;
  m = 4;
  ia = l + 1;
  ib = m + 1;
  ic = ia + ib - 1;

  /* Allocate memory */
  if (!(aa = NAG_ALLOC(ia, double)) ||
      !(bb = NAG_ALLOC(ib, double)) ||
      !(cc = NAG_ALLOC(ic, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  cc[0] = 1.0;
  for (i = 1; i <= ic - 1; ++i)
    cc[i] = cc[i-1] / (double) i;

  /* nag_1d_pade (e02rac).
   * Pade-approximants
   */
  nag_1d_pade(ia, ib, cc, aa, bb, &fail);
```

```
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_pade (e02rac).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  printf("\n");
  printf("    x        Pade           True\n");

  for (i = 0; i < 10; ++i)
    {
      x = (double)(i + 1) / 10.0;
      /* nag_1d_pade_eval (e02rbc).
       * Evaluation of fitted rational function as computed by
       * nag_1d_pade (e02rac)
       */
      nag_1d_pade_eval(aa, ia, bb, ib, x, &ans, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_1d_pade_eval (e02rbc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }

      tval = exp(x);
      printf("%6.1f%15.5e%15.5e\n", x, ans, tval);
    }

 END:
  NAG_FREE(aa);
  NAG_FREE(bb);
  NAG_FREE(cc);

  return exit_status;
}
```

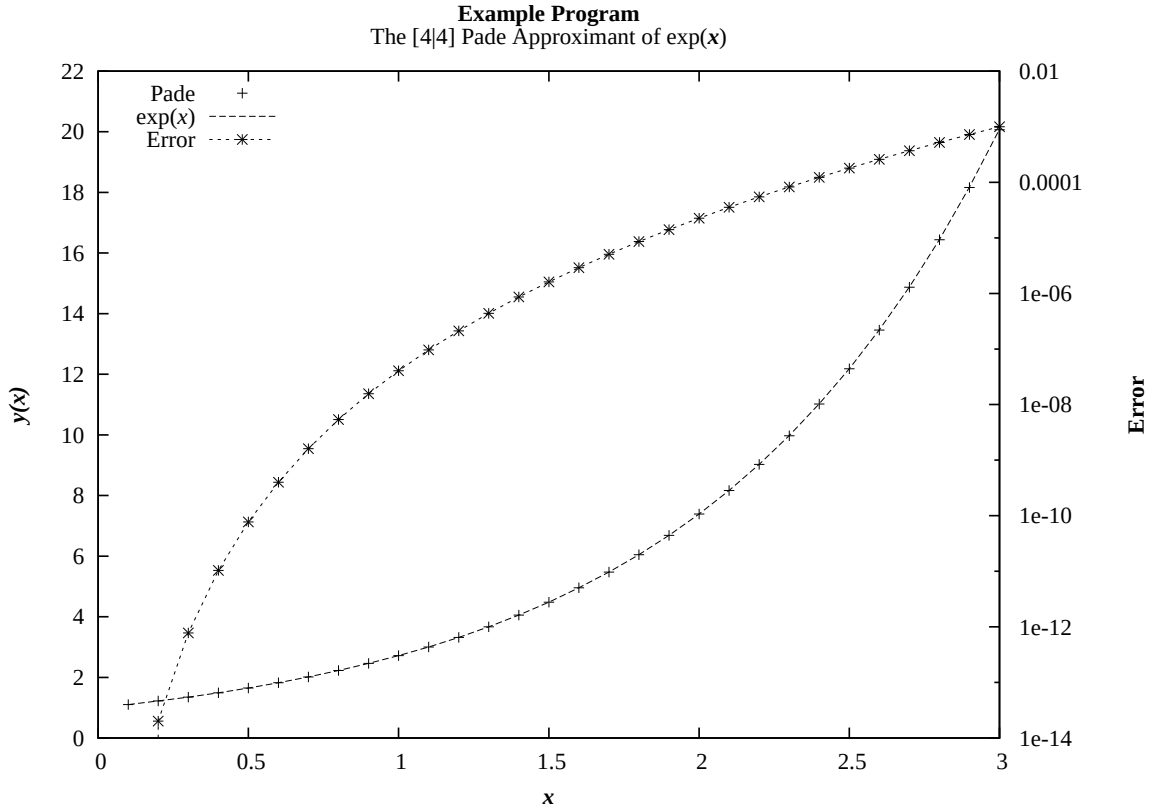
10.2 Program Data

None.

10.3 Program Results

```
nag_1d_pade_eval (e02rbc) Example Program Results

    x        Pade           True
   0.1    1.10517e+00    1.10517e+00
   0.2    1.22140e+00    1.22140e+00
   0.3    1.34986e+00    1.34986e+00
   0.4    1.49182e+00    1.49182e+00
   0.5    1.64872e+00    1.64872e+00
   0.6    1.82212e+00    1.82212e+00
   0.7    2.01375e+00    2.01375e+00
   0.8    2.22554e+00    2.22554e+00
   0.9    2.45960e+00    2.45960e+00
   1.0    2.71828e+00    2.71828e+00
```


Example Program
The [4|4] Pade Approximant of exp(x)
Pade
exp(x)
Error
y(x)
Error
x
0
2
4
6
8
10
12
14
16
18
20
22
0.5
1
1.5
2.5
3
0.01
0.0001
1e-06
1e-08
1e-10
1e-12
1e-14