NAG Library Function Document

nag_pde_bs_1d (d03ncc)

1 Purpose

nag_pde_bs_1d (d03ncc) solves the Black–Scholes equation for financial option pricing using a finite difference scheme.

2 Specification

```
#include <nag.h>
#include <nagd03.h>
```

```
void nag_pde_bs_1d (Nag_OptionType kopt, double x, Nag_MeshType mesh,
     Integer ns, double s[], Integer nt, double t[],
     const Nag_Boolean tdpar[], const double r[], const double q[],
     const double sigma[], double alpha, Integer ntkeep, double f[],
     double theta[], double delta[], double gamma[], double lambda[],
     double rho[], NagError *fail)
```

3 Description

nag_pde_bs_1d (d03ncc) solves the Black–Scholes equation (see Hull (1989) and Wilmott *et al.* (1995))

$$\frac{\partial f}{\partial t}+(r-q)S\frac{\partial f}{\partial S}+\frac{\sigma^2S^2}{2}\frac{\partial^2 f}{\partial S^2}=rf \tag{1}$$

$$S_{\text{min}}<S<S_{\text{max}}\,,\quad t_{\text{min}}<t<t_{\text{max}}\,, \tag{2}$$

for the value f of a European or American, put or call stock option, with exercise price X. In equation (1) t is time, S is the stock price, r is the risk free interest rate, q is the continuous dividend, and σ is the stock volatility. According to the values in the array **tdpar**, the arguments r, q and σ may each be either constant or functions of time. The function also returns values of various Greeks.

nag_pde_bs_1d (d03ncc) uses a finite difference method with a choice of time-stepping schemes. The method is explicit for **alpha** $=0.0$ and implicit for nonzero values of **alpha**. Second order time accuracy can be obtained by setting **alpha** $=0.5$. According to the value of the argument **mesh** the finite difference mesh may be either uniform, or user-defined in both S and t directions.

4 References

Hull J (1989) *Options, Futures and Other Derivative Securities* Prentice–Hall

Wilmott P, Howison S and Dewynne J (1995) *The Mathematics of Financial Derivatives* Cambridge University Press

5 Arguments

1: **kopt** – Nag_OptionType *Input*

On entry: specifies the kind of option to be valued.

kopt = Nag_EuropeanCall
A European call option.

kopt = Nag_AmericanCall
An American call option.

kopt = Nag_EuropeanPut
A European put option.

kopt = Nag_AmericanPut
An American put option.

Constraint: **kopt** = Nag_EuropeanCall, Nag_AmericanCall, Nag_EuropeanPut or Nag_AmericanPut.

2: **x** – double *Input*

On entry: the exercise price X.

3: **mesh** – Nag_MeshType *Input*

On entry: indicates the type of finite difference mesh to be used:

mesh = Nag_UniformMesh
Uniform mesh.

mesh = Nag_CustomMesh
Custom mesh supplied by you.

Constraint: **mesh** = Nag_UniformMesh or Nag_CustomMesh.

4: **ns** – Integer *Input*

On entry: the number of stock prices to be used in the finite difference mesh.

Constraint: **ns** ≥ 2.

5: **s**[**ns**] – double *Input/Output*

On entry: if **mesh** = Nag_CustomMesh, **s**$[i-1]$ must contain the ith stock price in the mesh, for $i = 1, 2, \ldots,$ **ns**. These values should be in increasing order, with **s**$[0] = S_{\text{min}}$ and **s**$[$**ns**$- 1] = S_{\text{max}}$.

If **mesh** = Nag_UniformMesh, **s**$[0]$ must be set to S_{min} and **s**$[$**ns**$- 1]$ to S_{max}, but **s**$[1]$, **s**$[2], \ldots,$ **s**$[$**ns**$- 2]$ need not be initialized, as they will be set internally by the function in order to define a uniform mesh.

On exit: if **mesh** = Nag_UniformMesh, the elements of **s** define a uniform mesh over $[S_{\text{min}}, S_{\text{max}}]$.

If **mesh** = Nag_CustomMesh, the elements of **s** are unchanged.

Constraints:

if **mesh** = Nag_CustomMesh, **s**$[0] \geq 0.0$ and **s**$[i-1] <$ **s**$[i]$, for $i = 1, 2, \ldots,$ **ns**$- 1$;
if **mesh** = Nag_UniformMesh, $0.0 \leq$ **s**$[0] <$ **s**$[$**ns**$- 1]$.

6: **nt** – Integer *Input*

On entry: the number of time-steps to be used in the finite difference method.

Constraint: **nt** ≥ 2.

7: **t**[**nt**] – double *Input/Output*

On entry: if **mesh** = Nag_CustomMesh then **t**$[j-1]$ must contain the jth time in the mesh, for $j = 1, 2, \ldots,$ **nt**. These values should be in increasing order, with **t**$[0] = t_{\text{min}}$ and **t**$[$**nt**$- 1] = t_{\text{max}}$.

If **mesh** = Nag_UniformMesh then **t**$[0]$ must be set to t_{min} and **t**$[$**nt**$- 1]$ to t_{max}, but **t**$[1]$, **t**$[2], \ldots,$ **t**$[$**nt**$- 2]$ need not be initialized, as they will be set internally by the function in order to define a uniform mesh.

On exit: if **mesh** = Nag_UniformMesh, the elements of **t** define a uniform mesh over $[t_{\text{min}}, t_{\text{max}}]$.

If **mesh** = Nag_CustomMesh, the elements of **t** are unchanged.

Constraints:

if **mesh** = Nag_CustomMesh, $\mathbf{t}[0] \geq 0.0$ and $\mathbf{t}[j-1] < \mathbf{t}[j]$, for $j = 1, 2, \ldots, \mathbf{nt} - 1$;
if **mesh** = Nag_UniformMesh, $0.0 \leq \mathbf{t}[0] < \mathbf{t}[\mathbf{nt} - 1]$.

8: **tdpar**[**3**] – const Nag_Boolean *Input*

On entry: specifies whether or not various arguments are time-dependent. More precisely, r is time-dependent if **tdpar**[0] = Nag_TRUE and constant otherwise. Similarly, **tdpar**[1] specifies whether q is time-dependent and **tdpar**[2] specifies whether σ is time-dependent.

9: **r**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **r** must be at least

nt when **tdpar**[0] = Nag_TRUE;
1 otherwise.

On entry: if **tdpar**[0] = Nag_TRUE then **r**$[j-1]$ must contain the value of the risk-free interest rate $r(t)$ at the jth time in the mesh, for $j = 1, 2, \ldots, \mathbf{nt}$.

If **tdpar**[0] = Nag_FALSE then **r**[0] must contain the constant value of the risk-free interest rate r. The remaining elements need not be set.

10: **q**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **q** must be at least

nt when **tdpar**[1] = Nag_TRUE;
1 otherwise.

On entry: if **tdpar**[1] = Nag_TRUE then **q**$[j-1]$ must contain the value of the continuous dividend $q(t)$ at the jth time in the mesh, for $j = 1, 2, \ldots, \mathbf{nt}$.

If **tdpar**[1] = Nag_FALSE then **q**[0] must contain the constant value of the continuous dividend q. The remaining elements need not be set.

11: **sigma**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **sigma** must be at least

nt when **tdpar**[2] = Nag_TRUE;
1 otherwise.

On entry: if **tdpar**[2] = Nag_TRUE then **sigma**$[j-1]$ must contain the value of the volatility $\sigma(t)$ at the jth time in the mesh, for $j = 1, 2, \ldots, \mathbf{nt}$.

If **tdpar**[2] = Nag_FALSE then **sigma**[0] must contain the constant value of the volatility σ. The remaining elements need not be set.

12: **alpha** – double *Input*

On entry: the value of λ to be used in the time-stepping scheme. Typical values include:

alpha = 0.0
Explicit forward Euler scheme.

alpha = 0.5
Implicit Crank–Nicolson scheme.

alpha = 1.0
Implicit backward Euler scheme.

The value 0.5 gives second-order accuracy in time. Values greater than 0.5 give unconditional stability. Since 0.5 is at the limit of unconditional stability this value does not damp oscillations.

Suggested value: **alpha** = 0.55.

Constraint: $0.0 \leq \mathbf{alpha} \leq 1.0$.

13: **ntkeep** – Integer *Input*

On entry: the number of solutions to be stored in the time direction. The function calculates the solution backwards from **t**[**nt** − 1] to **t**[0] at all times in the mesh. These time solutions and the corresponding Greeks will be stored at times **t**[$i - 1$], for $i = 1, 2, \ldots,$ **ntkeep**, in the arrays **f**, **theta**, **delta**, **gamma**, **lambda** and **rho**. Other time solutions will be discarded. To store all time solutions set **ntkeep** = **nt**.

Constraint: $1 \leq$ **ntkeep** $\leq$ **nt**.

14: **f**[**ns** × **ntkeep**] – double *Output*

On exit: **f**[**ns** × $(j - 1) + i - 1$], for $i = 1, 2, \ldots,$ **ns** and $j = 1, 2, \ldots,$ **ntkeep**, contains the value f of the option at the ith mesh point **s**[$i - 1$] at time **t**[$j - 1$].

15: **theta**[**ns** × **ntkeep**] – double *Output*
16: **delta**[**ns** × **ntkeep**] – double *Output*
17: **gamma**[**ns** × **ntkeep**] – double *Output*
18: **lambda**[**ns** × **ntkeep**] – double *Output*
19: **rho**[**ns** × **ntkeep**] – double *Output*

On exit: the values of various Greeks at the ith mesh point **s**[$i - 1$] at time **t**[$j - 1$], as follows:

$$\textbf{theta}[\textbf{ns} \times (j-1) + i - 1] = \frac{\partial f}{\partial t}, \quad \textbf{delta}[\textbf{ns} \times (j-1) + i - 1] = \frac{\partial f}{\partial S}, \quad \textbf{gamma}[\textbf{ns} \times (j-1) + i - 1] = \frac{\partial^2 f}{\partial S^2},$$

$$\textbf{lambda}[\textbf{ns} \times (j-1) + i - 1] = \frac{\partial f}{\partial \sigma}, \quad \textbf{rho}[\textbf{ns} \times (j-1) + i - 1] = \frac{\partial f}{\partial r}.$$

20: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **ns** = ⟨*value*⟩.
Constraint: **ns** ≥ 2.

On entry, **nt** = ⟨*value*⟩.
Constraint: **nt** ≥ 2.

On entry, **ntkeep** = ⟨*value*⟩.
Constraint: **ntkeep** ≥ 1.

NE_INT_2

On entry, **ntkeep** = ⟨*value*⟩ and **nt** = ⟨*value*⟩.
Constraint: **ntkeep** $\leq$ **nt**.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_NOT_STRICTLY_INCREASING

On entry, $\mathbf{s}[I] \le \mathbf{s}[I-1]$ in custom mesh: $I = \langle value\rangle$.

On entry, $\mathbf{t}[J] \le \mathbf{t}[J-1]$ in custom mesh: $J = \langle value\rangle$.

NE_REAL

On entry, **alpha** $= \langle value\rangle$.
Constraint: **alpha** ≤ 1.0.

On entry, **alpha** $= \langle value\rangle$.
Constraint: **alpha** ≥ 0.0.

On entry, $\mathbf{s}[0] < 0.0$: $\mathbf{s}[0] = \langle value\rangle$.

On entry, $\mathbf{t}[0] < 0.0$: $\mathbf{t}[0] = \langle value\rangle$.

NE_REAL_2

On entry, $\mathbf{s}[\mathbf{ns} - 1] = \langle value\rangle$ and $\mathbf{s}[0] = \langle value\rangle$.
Constraint: $\mathbf{s}[\mathbf{ns} - 1] > \mathbf{s}[0]$.

On entry, $\mathbf{t}[\mathbf{nt} - 1] = \langle value\rangle$ and $\mathbf{t}[0] = \langle value\rangle$.
Constraint: $\mathbf{t}[\mathbf{nt} - 1] > \mathbf{t}[0]$.

7 Accuracy

The accuracy of the solution f and the various derivatives returned by the function is dependent on the values of **ns** and **nt** supplied, the distribution of the mesh points, and the value of **alpha** chosen. For most choices of **alpha** the solution has a truncation error which is second-order accurate in S and first order accurate in t. For **alpha** $= 0.5$ the truncation error is also second-order accurate in t.

The simplest approach to improving the accuracy is to increase the values of both **ns** and **nt**.

8 Parallelism and Performance

nag_pde_bs_1d (d03ncc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_pde_bs_1d (d03ncc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

9.1 Timing

Each time-step requires the construction and solution of a tridiagonal system of linear equations. To calculate each of the derivatives **lambda** and **rho** requires a repetition of the entire solution process. The time taken for a call to the function is therefore proportional to **ns** $\times$ **nt**.

9.2 Algorithmic Details

nag_pde_bs_1d (d03ncc) solves equation (1) using a finite difference method. The solution is computed backwards in time from $t_{\max}$ to $t_{\min}$ using a λ scheme, which is implicit for all nonzero values of λ, and is unconditionally stable for values of $\lambda > 0.5$. For each time-step a tridiagonal system is constructed and solved to obtain the solution at the earlier time. For the explicit scheme ($\lambda = 0$) this tridiagonal system degenerates to a diagonal matrix and is solved trivially. For American options the solution at each time-step is inspected to check whether early exercise is beneficial, and amended accordingly.

To compute the arrays **lambda** and **rho**, which are derivatives of the stock value f with respect to the problem arguments σ and r respectively, the entire solution process is repeated with perturbed values of these arguments.

10 Example

This example, taken from Hull (1989), solves the one-dimensional Black–Scholes equation for valuation of a 5-month American put option on a non-dividend-paying stock with an exercise price of $50. The risk-free interest rate is 10% per annum, and the stock volatility is 40% per annum.

A fully implicit backward Euler scheme is used, with a mesh of 20 stock price intervals and 10 time intervals.

10.1 Program Text

```
/* nag_pde_bs_1d (d03ncc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <string.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd03.h>

#define F(I, J)      f[ns*((J) -1)+(I) -1]
#define THETA(I, J)  theta[ns*((J) -1)+(I) -1]
#define DELTA(I, J)  delta[ns*((J) -1)+(I) -1]
#define GAMMA(I, J)  gamma[ns*((J) -1)+(I) -1]
#define LAMBDA(I, J) lambda[ns*((J) -1)+(I) -1]
#define RHO(I, J)    rho[ns*((J) -1)+(I) -1]

int main(void)
{
  double      alpha, x;
  Integer     i, igreek, j, ns, nt, ntkeep, exit_status;
  double      *delta = 0, *f = 0, *gamma = 0, *lambda = 0, q[3], r[3],
              *rho = 0, *s = 0;
  double      sigma[3], *t = 0, *theta = 0, smin, smax, tmin, tmax;
  Nag_Boolean gprnt[5] = { Nag_TRUE, Nag_TRUE, Nag_TRUE, Nag_TRUE, Nag_TRUE };
  Nag_Boolean tdpar[3];
  const char  *gname[5] = { "Theta", "Delta", "Gamma", "Lambda", "Rho" };
  NagError    fail;

  INIT_FAIL(fail);

  printf("nag_pde_bs_1d (d03ncc) Example Program Results\n\n");

  /* Skip heading in data file */

#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
```

```
#endif
  exit_status = 0;

  /* Read problem parameters */

#ifdef _WIN32
  scanf_s("%lf", &x);
#else
  scanf("%lf", &x);
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%"NAG_IFMT"", &ns, &nt);
#else
  scanf("%"NAG_IFMT"%"NAG_IFMT"", &ns, &nt);
#endif
#ifdef _WIN32
  scanf_s("%lf%lf", &smin, &smax);
#else
  scanf("%lf%lf", &smin, &smax);
#endif
#ifdef _WIN32
  scanf_s("%lf%lf", &tmin, &tmax);
#else
  scanf("%lf%lf", &tmin, &tmax);
#endif
#ifdef _WIN32
  scanf_s("%lf", &alpha);
#else
  scanf("%lf", &alpha);
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"", &ntkeep);
#else
  scanf("%"NAG_IFMT"", &ntkeep);
#endif

  /* Allocate memory */

  if (!(s = NAG_ALLOC(ns, double)) ||
      !(t = NAG_ALLOC(nt, double)) ||
      !(f = NAG_ALLOC(ns*ntkeep, double)) ||
      !(theta = NAG_ALLOC(ns*ntkeep, double)) ||
      !(delta = NAG_ALLOC(ns*ntkeep, double)) ||
      !(gamma = NAG_ALLOC(ns*ntkeep, double)) ||
      !(lambda = NAG_ALLOC(ns*ntkeep, double)) ||
      !(rho = NAG_ALLOC(ns*ntkeep, double)))
    {
      printf("Allocation failure\n");
      exit_status = 1;
      goto END;
    }

  /* Set up input parameters for nag_pde_bs_1d (d03ncc) */

  s[0] = smin;
  s[ns-1] = smax;
  t[0] = tmin;
  t[nt-1] = tmax;
  tdpar[0] = Nag_FALSE;
  tdpar[1] = Nag_FALSE;
  tdpar[2] = Nag_FALSE;
  q[0] = 0.0;
  r[0] = 0.10;
  sigma[0] = 0.4;

  /* Call Black-Scholes solver */

  /* nag_pde_bs_1d (d03ncc).
   * Finite difference solution of the Black-Scholes equations
   */
  nag_pde_bs_1d(Nag_AmericanPut, x, Nag_UniformMesh, ns, s,
                nt, t, tdpar, r, q, sigma, alpha, ntkeep, f,
                theta, delta, gamma, lambda, rho, &fail);

  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_pde_bs_1d (d03ncc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Output option values */

  printf("\nOption Values\n");
  printf("-------------\n");
  printf("%14s  |  %s\n", "Stock Price", "Time to Maturity (months)");
  printf("%14s  | ", "");
  for (i = 0; i < ntkeep; i++) printf(" %13.4e", 12.0*(t[nt-1]-t[i]));
  printf("\n");
  for (i = 0; i < 74; i++) printf("-");
  printf("\n");
  for (i = 1; i <= ns; i++)
    {
      printf(" %13.4e  | ", s[i-1]);
      for (j = 1; j <= ntkeep; j++) printf(" %13.4e", F(i, j));
      printf("\n");
    }

  for (igreek = 0; igreek < 5; igreek++)
    {
      if (!gprnt[igreek]) continue;

      printf("\n%s\n", gname[igreek]);
      for (i = 0; i < (Integer) strlen(gname[igreek]); i++) printf("-");
      printf("\n%14s  |  %s\n", "Stock Price",
              "Time to Maturity (months)");
      printf("%14s  | ", "");
      for (i = 0; i < ntkeep; i++)
        printf(" %13.4e", 12.0*(t[nt-1]-t[i]));
      printf("\n");
      for (i = 0; i < 74; i++) printf("-");
      printf("\n");

      for (i = 1; i <= ns; i++)
        {
          printf(" %13.4e  | ", s[i-1]);
          switch (igreek)
            {
            case 0:
              for (j = 1; j <= ntkeep; j++)
                printf(" %13.4e", THETA(i, j));
              break;
            case 1:
              for (j = 1; j <= ntkeep; j++)
                printf(" %13.4e", DELTA(i, j));
              break;
            case 2:
              for (j = 1; j <= ntkeep; j++)
                printf(" %13.4e", GAMMA(i, j));
              break;
            case 3:
              for (j = 1; j <= ntkeep; j++)
                printf(" %13.4e", LAMBDA(i, j));
              break;
            case 4:
              for (j = 1; j <= ntkeep; j++)
                printf(" %13.4e", RHO(i, j));
              break;
            default:
              break;
            }
          printf("\n");
        }
    }

 END:
  NAG_FREE(s);
  NAG_FREE(t);
  NAG_FREE(f);
  NAG_FREE(theta);
  NAG_FREE(delta);
  NAG_FREE(gamma);
  NAG_FREE(lambda);
  NAG_FREE(rho);

  return exit_status;
}
```

10.2 Program Data

```
nag_pde_bs_1d (d03ncc) Example Program Data
  50.
  21  11
  0.0  100.
  0.0  0.4166667
  1.0
  4
```

10.3 Program Results

```
nag_pde_bs_1d (d03ncc) Example Program Results


Option Values
-------------
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
--------------------------------------------------------------------------
    0.0000e+00  |     5.0000e+01    5.0000e+01    5.0000e+01    5.0000e+01
    5.0000e+00  |     4.5000e+01    4.5000e+01    4.5000e+01    4.5000e+01
    1.0000e+01  |     4.0000e+01    4.0000e+01    4.0000e+01    4.0000e+01
    1.5000e+01  |     3.5000e+01    3.5000e+01    3.5000e+01    3.5000e+01
    2.0000e+01  |     3.0000e+01    3.0000e+01    3.0000e+01    3.0000e+01
    2.5000e+01  |     2.5000e+01    2.5000e+01    2.5000e+01    2.5000e+01
    3.0000e+01  |     2.0000e+01    2.0000e+01    2.0000e+01    2.0000e+01
    3.5000e+01  |     1.5000e+01    1.5000e+01    1.5000e+01    1.5000e+01
    4.0000e+01  |     1.0154e+01    1.0096e+01    1.0046e+01    1.0012e+01
    4.5000e+01  |     6.5848e+00    6.4424e+00    6.2916e+00    6.1306e+00
    5.0000e+01  |     4.0672e+00    3.8785e+00    3.6729e+00    3.4463e+00
    5.5000e+01  |     2.4264e+00    2.2423e+00    2.0454e+00    1.8336e+00
    6.0000e+01  |     1.4174e+00    1.2662e+00    1.1096e+00    9.4813e-01
    6.5000e+01  |     8.1951e-01    7.0724e-01    5.9532e-01    4.8515e-01
    7.0000e+01  |     4.7241e-01    3.9411e-01    3.1904e-01    2.4845e-01
    7.5000e+01  |     2.7257e-01    2.2016e-01    1.7174e-01    1.2815e-01
    8.0000e+01  |     1.5725e-01    1.2328e-01    9.2935e-02    6.6682e-02
    8.5000e+01  |     8.9662e-02    6.8478e-02    5.0100e-02    3.4731e-02
    9.0000e+01  |     4.8449e-02    3.6251e-02    2.5901e-02    1.7469e-02
    9.5000e+01  |     2.1100e-02    1.5584e-02    1.0968e-02    7.2680e-03
    1.0000e+02  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00

Theta
-----
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
--------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    4.0000e+01  |    -1.4043e+00   -1.1857e+00   -8.3285e-01   -2.8064e-01
    4.5000e+01  |    -3.4185e+00   -3.6183e+00   -3.8646e+00   -4.1880e+00
    5.0000e+01  |    -4.5285e+00   -4.9339e+00   -5.4387e+00   -6.0796e+00
    5.5000e+01  |    -4.4165e+00   -4.7277e+00   -5.0821e+00   -5.4821e+00
    6.0000e+01  |    -3.6294e+00   -3.7585e+00   -3.8748e+00   -3.9632e+00
    6.5000e+01  |    -2.6946e+00   -2.6860e+00   -2.6441e+00   -2.5561e+00
    7.0000e+01  |    -1.8790e+00   -1.8018e+00   -1.6941e+00   -1.5505e+00
    7.5000e+01  |    -1.2578e+00   -1.1621e+00   -1.0461e+00   -9.0969e-01
    8.0000e+01  |    -8.1539e-01   -7.2821e-01   -6.3006e-01   -5.2314e-01
    8.5000e+01  |    -5.0841e-01   -4.4106e-01   -3.6887e-01   -2.9433e-01
    9.0000e+01  |    -2.9276e-01   -2.4840e-01   -2.0237e-01   -1.5656e-01
    9.5000e+01  |    -1.3237e-01   -1.1079e-01   -8.8802e-02   -6.7378e-02
    1.0000e+02  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00

Delta
-----
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
----------------------------------------------------------------------
    0.0000e+00  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    5.0000e+00  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    1.0000e+01  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    1.5000e+01  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    2.0000e+01  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    2.5000e+01  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    3.0000e+01  |    -1.0000e+00   -1.0000e+00   -1.0000e+00   -1.0000e+00
    3.5000e+01  |    -9.8457e-01   -9.9042e-01   -9.9536e-01   -9.9883e-01
    4.0000e+01  |    -8.4152e-01   -8.5576e-01   -8.7084e-01   -8.8694e-01
    4.5000e+01  |    -6.0871e-01   -6.2173e-01   -6.3735e-01   -6.5654e-01
    5.0000e+01  |    -4.1584e-01   -4.2000e-01   -4.2463e-01   -4.2970e-01
    5.5000e+01  |    -2.6498e-01   -2.6123e-01   -2.5633e-01   -2.4982e-01
    6.0000e+01  |    -1.6069e-01   -1.5351e-01   -1.4500e-01   -1.3485e-01
    6.5000e+01  |    -9.4501e-02   -8.7208e-02   -7.9055e-02   -6.9969e-02
    7.0000e+01  |    -5.4694e-02   -4.8708e-02   -4.2358e-02   -3.5699e-02
    7.5000e+01  |    -3.1515e-02   -2.7084e-02   -2.2610e-02   -1.8177e-02
    8.0000e+01  |    -1.8291e-02   -1.5168e-02   -1.2164e-02   -9.3423e-03
    8.5000e+01  |    -1.0880e-02   -8.7026e-03   -6.7034e-03   -4.9214e-03
    9.0000e+01  |    -6.8562e-03   -5.2894e-03   -3.9132e-03   -2.7463e-03
    9.5000e+01  |    -4.8449e-03   -3.6251e-03   -2.5901e-03   -1.7469e-03
    1.0000e+02  |    -4.2199e-03   -3.1168e-03   -2.1936e-03   -1.4536e-03

Gamma
-----
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
----------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.5000e+01  |     6.1726e-03    3.8321e-03    1.8558e-03    4.6773e-04
    4.0000e+01  |     5.1047e-02    5.0031e-02    4.7953e-02    4.4288e-02
    4.5000e+01  |     4.2075e-02    4.3582e-02    4.5444e-02    4.7873e-02
    5.0000e+01  |     3.5072e-02    3.7109e-02    3.9646e-02    4.2863e-02
    5.5000e+01  |     2.5275e-02    2.6400e-02    2.7671e-02    2.9089e-02
    6.0000e+01  |     1.6442e-02    1.6688e-02    1.6860e-02    1.6900e-02
    6.5000e+01  |     1.0032e-02    9.8331e-03    9.5193e-03    9.0515e-03
    7.0000e+01  |     5.8907e-03    5.5669e-03    5.1595e-03    4.6562e-03
    7.5000e+01  |     3.3809e-03    3.0827e-03    2.7396e-03    2.3529e-03
    8.0000e+01  |     1.9091e-03    1.6834e-03    1.4388e-03    1.1808e-03
    8.5000e+01  |     1.0551e-03    9.0291e-04    7.4543e-04    5.8760e-04
    9.0000e+01  |     5.5449e-04    4.6239e-04    3.7065e-04    2.8244e-04
    9.5000e+01  |     2.5001e-04    2.0330e-04    1.5859e-04    1.1731e-04
    1.0000e+02  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00

Lambda
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   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
--------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    4.0000e+01  |     6.3243e+00    5.1893e+00    3.8089e+00    2.1118e+00
    4.5000e+01  |     1.0721e+01    9.9718e+00    9.2140e+00    8.4953e+00
    5.0000e+01  |     1.2381e+01    1.1807e+01    1.1228e+01    1.0636e+01
    5.5000e+01  |     1.1483e+01    1.0837e+01    1.0142e+01    9.3795e+00
    6.0000e+01  |     9.3227e+00    8.5840e+00    7.7870e+00    6.9211e+00
    6.5000e+01  |     6.9621e+00    6.2206e+00    5.4412e+00    4.6264e+00
    7.0000e+01  |     4.9268e+00    4.2651e+00    3.5937e+00    2.9227e+00
    7.5000e+01  |     3.3602e+00    2.8204e+00    2.2920e+00    1.7866e+00
    8.0000e+01  |     2.2221e+00    1.8126e+00    1.4248e+00    1.0683e+00
    8.5000e+01  |     1.4122e+00    1.1240e+00    8.5856e-01    6.2248e-01
    9.0000e+01  |     8.2686e-01    6.4587e-01    4.8252e-01    3.4083e-01
    9.5000e+01  |     3.7891e-01    2.9252e-01    2.1553e-01    1.4976e-01
    1.0000e+02  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00

Rho
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   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
--------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    1.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    2.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.0000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    3.5000e+01  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    4.0000e+01  |    -7.1918e+00   -6.0114e+00   -4.5204e+00   -2.5855e+00
    4.5000e+01  |    -8.4541e+00   -7.6378e+00   -6.8479e+00   -6.1657e+00
    5.0000e+01  |    -7.5988e+00   -6.9323e+00   -6.2879e+00   -5.6707e+00
    5.5000e+01  |    -5.8905e+00   -5.2837e+00   -4.6809e+00   -4.0772e+00
    6.0000e+01  |    -4.1854e+00   -3.6547e+00   -3.1306e+00   -2.6135e+00
    6.5000e+01  |    -2.8221e+00   -2.3904e+00   -1.9743e+00   -1.5775e+00
    7.0000e+01  |    -1.8437e+00   -1.5137e+00   -1.2055e+00   -9.2283e-01
    7.5000e+01  |    -1.1812e+00   -9.4071e-01   -7.2326e-01   -5.3162e-01
    8.0000e+01  |    -7.4513e-01   -5.7680e-01   -4.2921e-01   -3.0383e-01
    8.5000e+01  |    -4.5907e-01   -3.4659e-01   -2.5060e-01   -1.7161e-01
    9.0000e+01  |    -2.6550e-01   -1.9656e-01   -1.3892e-01   -9.2652e-02
    9.5000e+01  |    -1.2280e-01   -8.9807e-02   -6.2569e-02   -4.1033e-02
    1.0000e+02  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
```
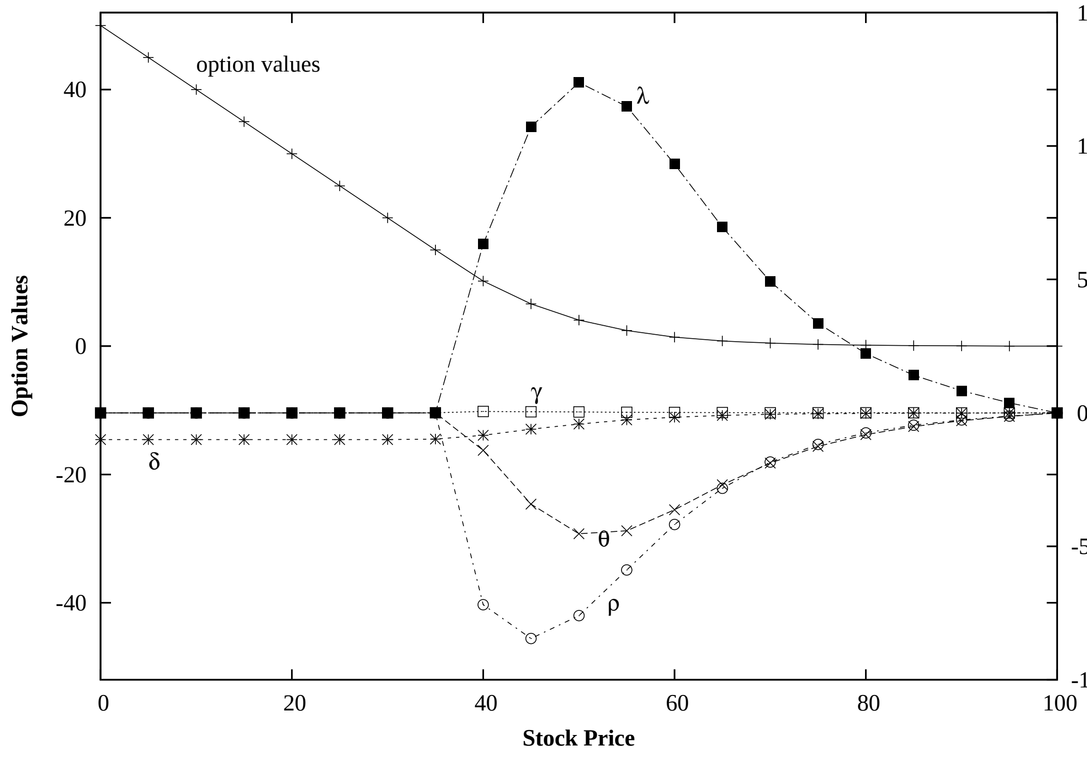
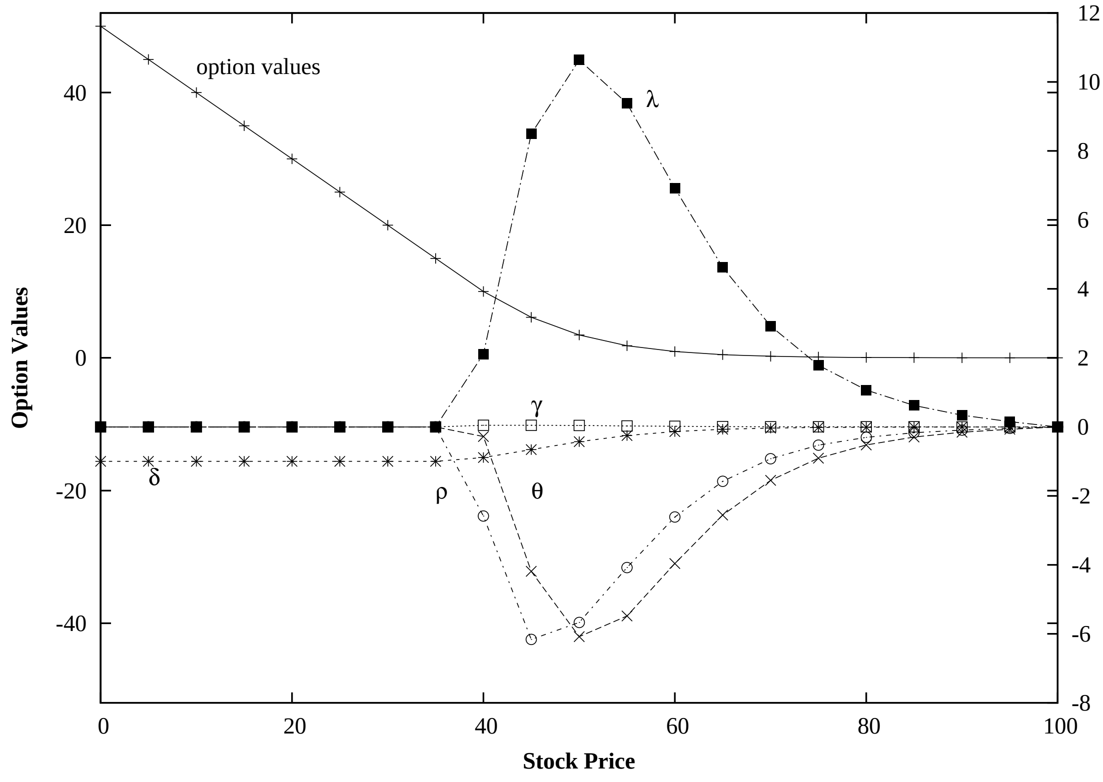

Example Program
Option Values and Derivatives at 5 Months to Maturity



Option Values and Derivatives at 3.5 Months to Maturity


option values
λ
γ
δ
ρ
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Option Values
Derivatives
Stock Price