NAG Library Function Document

nag_ode_bvp_ps_lin_coeffs (d02uac)

1 Purpose

nag_ode_bvp_ps_lin_coeffs (d02uac) obtains the Chebyshev coefficients of a function discretized on Chebyshev Gauss–Lobatto points. The set of discretization points on which the function is evaluated is usually obtained by a previous call to nag_ode_bvp_ps_lin_cgl_grid (d02ucc).

2 Specification

```
#include <nag.h>
#include <nagd02.h>
```

```
void nag_ode_bvp_ps_lin_coeffs (Integer n, const double f[], double c[],
     NagError *fail)
```

3 Description

nag_ode_bvp_ps_lin_coeffs (d02uac) computes the coefficients c_j, for $j = 1, 2, \ldots, n+1$, of the interpolating Chebyshev series

$$\tfrac{1}{2}c_1 T_0(\bar{x}) + c_2 T_1(\bar{x}) + c_3 T_2(\bar{x}) + \cdots + c_{n+1} T_n(\bar{x}),$$

which interpolates the function $f(x)$ evaluated at the Chebyshev Gauss–Lobatto points

$$\bar{x}_r = -\cos((r-1)\pi/n), \quad r = 1, 2, \ldots, n+1.$$

Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$ defined on $[-1, 1]$. In terms of your original variable, x say, the input values at which the function values are to be provided are

$$x_r = -\tfrac{1}{2}(b-a)\cos(\pi(r-1)/n) + \tfrac{1}{2}(b+a), \quad r = 1, 2, \ldots, n+1,$$

where b and a are respectively the upper and lower ends of the range of x over which the function is required.

4 References

Canuto C (1988) *Spectral Methods in Fluid Dynamics* 502 Springer

Canuto C, Hussaini M Y, Quarteroni A and Zang T A (2006) *Spectral Methods: Fundamentals in Single Domains* Springer

Trefethen L N (2000) *Spectral Methods in MATLAB* SIAM

5 Arguments

1: **n** – Integer *Input*

On entry: n, where the number of grid points is $n+1$. This is also the largest order of Chebyshev polynomial in the Chebyshev series to be computed.

Constraint: $\mathbf{n} > 0$ and $\mathbf{n}$ is even.

2: $\mathbf{f}[\mathbf{n}+\mathbf{1}]$ – const double *Input*

On entry: the function values $f(x_r)$, for $r = 1, 2, \ldots, n+1$.

3: **c**[**n** + **1**] – double *Output*

On exit: the Chebyshev coefficients, c_j, for $j = 1, 2, \ldots, n + 1$.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** > 1.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** is even.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The Chebyshev coefficients computed should be accurate to within a small multiple of ***machine precision***.

8 Parallelism and Performance

nag_ode_bvp_ps_lin_coeffs (d02uac) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_ode_bvp_ps_lin_coeffs (d02uac) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The number of operations is of the order $n\log(n)$ and the memory requirements are $O(n)$; thus the computation remains efficient and practical for very fine discretizations (very large values of n).

10 Example

See Section 10 in nag_ode_bvp_ps_lin_solve (d02uec).