

NAG Library Routine Document

S22AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S22AAF returns a sequence of values for either the unnormalized or normalized Legendre functions of the first kind $P_n^m(x)$ or $\overline{P}_n^m(x)$ for real x of a given order m and degree $n = 0, 1, \dots, N$.

2 Specification

SUBROUTINE S22AAF (MODE, X, M, NL, P, IFAIL)

INTEGER MODE, M, NL, IFAIL

REAL (KIND=nag_wp) X, P(0:NL)

3 Description

S22AAF evaluates a sequence of values for either the unnormalized or normalized Legendre ($m = 0$) or associated Legendre ($m \neq 0$) functions of the first kind $P_n^m(x)$ or $\overline{P}_n^m(x)$, where x is real with $-1 \leq x \leq 1$, of order m and degree $n = 0, 1, \dots, N$ defined by

$$P_n^m(x) = (1-x^2)^{m/2} \frac{d^m}{dx^m} P_n(x) \quad \text{if } m \geq 0,$$

$$P_n^m(x) = \frac{(n+m)!}{(n-m)!} P_n^{-m}(x) \quad \text{if } m < 0 \quad \text{and}$$

$$\overline{P}_n^m(x) = \sqrt{\frac{(2n+1)(n-m)!}{2(n+m)!}} P_n^m(x)$$

respectively; $P_n(x)$ is the (unassociated) Legendre polynomial of degree n given by

$$P_n(x) \equiv P_n^0(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

(the *Rodrigues formula*). Note that some authors (e.g., Abramowitz and Stegun (1972)) include an additional factor of $(-1)^m$ (the *Condon-Shortley Phase*) in the definitions of $P_n^m(x)$ and $\overline{P}_n^m(x)$. They use the notation $P_{mn}(x) \equiv (-1)^m P_n^m(x)$ in order to distinguish between the two cases.

S22AAF is based on a standard recurrence relation described in Section 8.5.3 of Abramowitz and Stegun (1972). Constraints are placed on the values of m and n in order to avoid the possibility of machine overflow. It also sets the appropriate elements of the array P (see Section 5) to zero whenever the required function is not defined for certain values of m and n (e.g., $m = -5$ and $n = 3$).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

- 1: MODE – INTEGER *Input*
On entry: indicates whether the sequence of function values is to be returned unnormalized or normalized.
 MODE = 1
 The sequence of function values is returned unnormalized.
 MODE = 2
 The sequence of function values is returned normalized.
Constraint: MODE = 1 or 2.
- 2: X – REAL (KIND=nag_wp) *Input*
On entry: the argument x of the function.
Constraint: $\text{abs}(X) \leq 1.0$.
- 3: M – INTEGER *Input*
On entry: the order m of the function.
Constraint: $\text{abs}(M) \leq 27$.
- 4: NL – INTEGER *Input*
On entry: the degree N of the last function required in the sequence.
Constraints:
 NL \geq 0;
 if M = 0, NL \leq 100;
 if M \neq 0, NL \leq 55 – abs(M).
- 5: P(0 : NL) – REAL (KIND=nag_wp) array *Output*
On exit: the required sequence of function values as follows:
 if MODE = 1, P(n) contains $F_n^m(x)$, for $n = 0, 1, \dots, N$;
 if MODE = 2, P(n) contains $\overline{F}_n^m(x)$, for $n = 0, 1, \dots, N$.
- 6: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, –1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value –1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value –1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $\text{abs}(X) > 1.0$,
 or $\text{MODE} \neq 1$ or 2 ,
 or $NL < 0$,
 or $NL > 100$ when $M = 0$,
 or $\text{abs}(M) > 27$,
 or $NL + \text{abs}(M) > 55$ when $M \neq 0$.

7 Accuracy

The computed function values should be accurate to within a small multiple of the *machine precision* except when underflow (or overflow) occurs, in which case the true function values are within a small multiple of the underflow (or overflow) threshold of the machine.

8 Further Comments

None.

9 Example

This example reads the values of the arguments x , m and N from a file, calculates the sequence of unnormalized associated Legendre function values $P_n^m(x), P_{n+1}^m(x), \dots, P_{n+N}^m(x)$, and prints the results.

9.1 Program Text

```

Program s22aafe
!      S22AAF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
!      Use nag_library, Only: nag_wp, s22aaf
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nlmax = 100, nout = 6
!      .. Local Scalars ..
!      Real (Kind=nag_wp)          :: x
!      Integer                     :: ifail, m, mode, n, nl
!      .. Local Arrays ..
!      Real (Kind=nag_wp)          :: p(0:nlmax)
!      .. Executable Statements ..
!      Write (nout,*) 'S22AAF Example Program Results'
!
!      Skip heading in data file
!      Read (nin,*)
!
!      Read (nin,*) mode, x, m, nl
!
!      Write (nout,*)
!
!      ifail = 0
!      Call s22aaf(mode,x,m,nl,p,ifail)
!
!      Write (nout,*) 'MODE      X      M      NL'
!      Write (nout,*)

```

```

Write (nout,99999) mode, x, m, nl
Write (nout,*)

Select Case (mode)
Case (1)

  If (m==0) Then
    Write (nout,*) 'Unnormalized Legendre function values'
  Else
    Write (nout,*) 'Unnormalized associated Legendre function values'
  End If

Case (2)

  If (m==0) Then
    Write (nout,*) 'Normalized Legendre function values'
  Else
    Write (nout,*) 'Normalized associated Legendre function values'
  End If

End Select

Write (nout,*) ' n      P(n)'

Do n = 0, nl
  Write (nout,99998) n, p(n)
End Do

99999 Format (1X,I3,4X,F5.1,2I6)
99998 Format (1X,I2,1X,1P,E12.4)
End Program s22aafe

```

9.2 Program Data

S22AAF Example Program Data

1 0.5 2 3 : Values of MODE, X, M and NL

9.3 Program Results

S22AAF Example Program Results

| MODE | X | M | NL |
|------|-----|---|----|
| 1 | 0.5 | 2 | 3 |

Unnormalized associated Legendre function values

| n | P(n) |
|---|------------|
| 0 | 0.0000E+00 |
| 1 | 0.0000E+00 |
| 2 | 2.2500E+00 |
| 3 | 5.6250E+00 |
