

NAG Library Routine Document

S19ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S19ABF returns a value for the Kelvin function $\text{bei } x$ via the function name.

2 Specification

```
FUNCTION S19ABF (X, IFAIL)
REAL (KIND=nag_wp) S19ABF
INTEGER IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S19ABF evaluates an approximation to the Kelvin function $\text{bei } x$.

Note: $\text{bei}(-x) = \text{bei } x$, so the approximation need only consider $x \geq 0.0$.

The routine is based on several Chebyshev expansions:

For $0 \leq x \leq 5$,

$$\text{bei } x = \frac{x^2}{4} \sum_{r=0}^l a_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{5}\right)^4 - 1;$$

For $x > 5$,

$$\begin{aligned} \text{bei } x = & \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \left[\left(1 + \frac{1}{x}a(t)\right) \sin \alpha - \frac{1}{x}b(t) \cos \alpha \right] \\ & + \frac{e^{x/\sqrt{2}}}{\sqrt{2\pi x}} \left[\left(1 + \frac{1}{x}c(t)\right) \cos \beta - \frac{1}{x}d(t) \sin \beta \right] \end{aligned}$$

where $\alpha = \frac{x}{\sqrt{2}} - \frac{\pi}{8}$, $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$,

and $a(t)$, $b(t)$, $c(t)$, and $d(t)$ are expansions in the variable $t = \frac{10}{x} - 1$.

When x is sufficiently close to zero, the result is computed as $\text{bei } x = \frac{x^2}{4}$. If this result would underflow, the result returned is $\text{bei } x = 0.0$.

For large x , there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the routine must fail.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $\text{abs}(X)$ is too large for an accurate result to be returned. On soft failure, the routine returns zero. See also the Users' Note for your implementation.

7 Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let E be the absolute error in the function, and δ be the relative error in the argument. If δ is somewhat larger than the *machine precision*, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (-\text{ber}_1 x + \text{bei}_1 x) \right| \delta$$

(provided E is within machine bounds).

For small x the error amplification is insignificant and thus the absolute error is effectively bounded by the *machine precision*.

For medium and large x , the error behaviour is oscillatory and its amplitude grows like $\sqrt{\frac{x}{2\pi}} e^{x/\sqrt{2}}$.

Therefore it is impossible to calculate the functions with any accuracy when $\sqrt{x} e^{x/\sqrt{2}} > \frac{\sqrt{2\pi}}{\delta}$. Note that this value of x is much smaller than the minimum value of x for which the function overflows.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```

Program s19abfe

!      S19ABF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
!      Use nag_library, Only: nag_wp, s19abf
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Real (Kind=nag_wp)         :: x, y
!      Integer                     :: ifail, ioerr
!      .. Executable Statements ..
!      Write (nout,*) 'S19ABF Example Program Results'

!      Skip heading in data file
!      Read (nin,*)

!
!      Write (nout,*)
!      Write (nout,*) '      X      Y'
!      Write (nout,*)

data: Do
!      Read (nin,*,Iostat=ioerr) x

!      If (ioerr<0) Then
!          Exit data
!      End If

!      ifail = -1
!      y = s19abf(x,ifail)

!      If (ifail<0) Then
!          Exit data
!      End If

!      Write (nout,99999) x, y
!      End Do data

99999 Format (1X,1P,2E12.3)
End Program s19abfe

```

9.2 Program Data

```

S19ABF Example Program Data
0.1
1.0
2.5
5.0
10.0
15.0
-1.0

```

9.3 Program Results

```

S19ABF Example Program Results

      X      Y
1.000E-01  2.500E-03
1.000E+00  2.496E-01

```

2.500E+00	1.457E+00
5.000E+00	1.160E-01
1.000E+01	5.637E+01
1.500E+01	-2.953E+03
-1.000E+00	2.496E-01
