NAG Library Routine Document

S10AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S10AAF returns a value for the hyperbolic tangent, $\tanh x$, via the function name.

2 Specification

```
FUNCTION S10AAF (X, IFAIL)
REAL (KIND=nag_wp) S10AAF
INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S10AAF calculates an approximate value for the hyperbolic tangent of its argument, $\tanh x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\tanh x = x \times y(t) = x{\sum_{r=0}}' a_r T_r(t)$$

where $-1 \leq x \leq 1$, $-1 \leq t \leq 1$, and $t = 2x^2 - 1$.

For $1 < |x| < E_1$ (see the Users' Note for your implementation for value of E_1)

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For $|x| \geq E_1$, $\tanh x = \text{sign}\, x$ to within the representation accuracy of the machine and so this approximation is used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left|\frac{2x}{\sinh 2x}\delta\right|.$$

That is, a relative error in the argument, x, is amplified by a factor approximately $\frac{2x}{\sinh 2x}$, in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

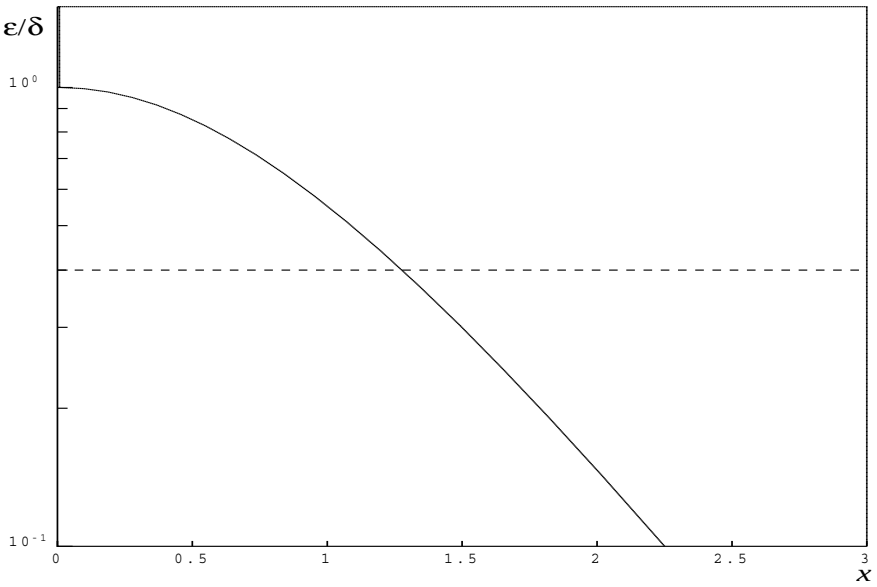

Figure 1

It should be noted that this factor is always less than or equal to 1.0 and away from $x = 0$ the accuracy will eventually be limited entirely by the precision of machine representation.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
    Program s10aafe

!     S10AAF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s10aaf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S10AAF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           Y'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = 0
        y = s10aaf(x,ifail)

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s10aafe
```

9.2 Program Data

```
S10AAF Example Program Data
          -20.0
           -5.0
            0.5
            5.0
```

9.3 Program Results

```
 S10AAF Example Program Results

      X           Y

   -2.000E+01  -1.000E+00
   -5.000E+00  -9.999E-01
    5.000E-01   4.621E-01
    5.000E+00   9.999E-01
```