NAG Library Routine Document

F08NSF (ZGEHRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08NSF (ZGEHRD) reduces a complex general matrix to Hessenberg form.

2 Specification

```
SUBROUTINE F08NSF (N, ILO, IHI, A, LDA, TAU, WORK, LWORK, INFO)

INTEGER                N, ILO, IHI, LDA, LWORK, INFO
COMPLEX (KIND=nag_wp)  A(LDA,*), TAU(*), WORK(max(1,LWORK))
```

The routine may be called by its LAPACK name ***zgehrd***.

3 Description

F08NSF (ZGEHRD) reduces a complex general matrix A to upper Hessenberg form H by a unitary similarity transformation: $A = QHQ^{\mathrm{H}}$. H has real subdiagonal elements.

The matrix Q is not formed explicitly, but is represented as a product of elementary reflectors (see the F08 Chapter Introduction for details). Routines are provided to work with Q in this representation (see Section 8).

The routine can take advantage of a previous call to F08NVF (ZGEBAL), which may produce a matrix with the structure:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{pmatrix}$$

where A_{11} and A_{33} are upper triangular. If so, only the central diagonal block A_{22}, in rows and columns i_{lo} to i_{hi}, needs to be reduced to Hessenberg form (the blocks A_{12} and A_{23} will also be affected by the reduction). Therefore the values of i_{lo} and i_{hi} determined by F08NVF (ZGEBAL) can be supplied to the routine directly. If F08NVF (ZGEBAL) has not previously been called however, then i_{lo} must be set to 1 and i_{hi} to n.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: ILO – INTEGER *Input*
3: IHI – INTEGER *Input*

On entry: if A has been output by F08NVF (ZGEBAL), then ILO and IHI **must** contain the values returned by that routine. Otherwise, ILO must be set to 1 and IHI to N.

Constraints:

if $N > 0$, $1 \leq ILO \leq IHI \leq N$;
if $N = 0$, $ILO = 1$ and $IHI = 0$.

4: A(LDA,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, N)$.

On entry: the n by n general matrix A.

On exit: A is overwritten by the upper Hessenberg matrix H and details of the unitary matrix Q. The subdiagonal elements of H are real.

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08NSF (ZGEHRD) is called.

Constraint: $LDA \geq \max(1, N)$.

6: TAU($*$) – COMPLEX (KIND=nag_wp) array *Output*

Note: the dimension of the array TAU must be at least $\max(1, N - 1)$.

On exit: further details of the unitary matrix Q.

7: WORK($\max(1, LWORK)$) – COMPLEX (KIND=nag_wp) array *Workspace*

On exit: if $INFO = 0$, the real part of WORK(1) contains the minimum value of LWORK required for optimal performance.

8: LWORK – INTEGER *Input*

On entry: the dimension of the array WORK as declared in the (sub)program from which F08NSF (ZGEHRD) is called.

If $LWORK = -1$, a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued.

Suggested value: for optimal performance, $LWORK \geq N \times nb$, where nb is the optimal ***block size***.

Constraint: $LWORK \geq \max(1, N)$ or $LWORK = -1$.

9: INFO – INTEGER *Output*

On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If $INFO = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed Hessenberg matrix H is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of H themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues, eigenvectors or Schur factorization.

8 Further Comments

The total number of real floating point operations is approximately $\frac{8}{3}q^2(2q+3n)$, where $q = i_{\mathrm{hi}} - i_{\mathrm{lo}}$; if $i_{\mathrm{lo}} = 1$ and $i_{\mathrm{hi}} = n$, the number is approximately $\frac{40}{3}n^3$.

To form the unitary matrix Q F08NSF (ZGEHRD) may be followed by a call to F08NTF (ZUNGHR):

```
CALL ZUNGHR(N,ILO,IHI,A,LDA,TAU,WORK,LWORK,INFO)
```

To apply Q to an m by n complex matrix C F08NSF (ZGEHRD) may be followed by a call to F08NUF (ZUNMHR). For example,

```
CALL ZUNMHR('Left','No Transpose',M,N,ILO,IHI,A,LDA,TAU,C,LDC, &
            WORK,LWORK,INFO)
```

forms the matrix product QC.

The real analogue of this routine is F08NEF (DGEHRD).

9 Example

This example computes the upper Hessenberg form of the matrix A, where

$$A = \begin{pmatrix} -3.97 - 5.04i & -4.11 + 3.70i & -0.34 + 1.01i & 1.29 - 0.86i \\ 0.34 - 1.50i & 1.52 - 0.43i & 1.88 - 5.38i & 3.36 + 0.65i \\ 3.31 - 3.85i & 2.50 + 3.45i & 0.88 - 1.08i & 0.64 - 1.48i \\ -1.10 + 0.82i & 1.81 - 1.59i & 3.25 + 1.33i & 1.57 - 3.44i \end{pmatrix}.$$

9.1 Program Text

```
    Program f08nsfe

!     F08NSF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zgehrd
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Complex (Kind=nag_wp), Parameter :: zero = (0.0E0_nag_wp,0.0E0_nag_wp)
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, lwork, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: a(:,:), tau(:), work(:)
      Character (1)                    :: clabs(1), rlabs(1)
!     .. Executable Statements ..
      Write (nout,*) 'F08NSF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      lwork = 64*n
      Allocate (a(lda,n),tau(n-1),work(lwork))

!     Read A from data file

      Read (nin,*)(a(i,1:n),i=1,n)

!     Reduce A to upper Hessenberg form

!     The NAG name equivalent of zgehrd is f08nsf
```

```
      Call zgehrd(n,1,n,a,lda,tau,work,lwork,info)

!     Set the elements below the first sub-diagonal to zero

      Do i = 1, n - 2
        a(i+2:n,i) = zero
      End Do

!     Print upper Hessenberg form

      Write (nout,*)
      Flush (nout)

!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04dbf('General',' ',n,n,a,lda,'Bracketed','F7.4', &
        'Upper Hessenberg form','Integer',rlabs,'Integer',clabs,80,0,ifail)

    End Program f08nsfe
```

9.2 Program Data

```
F08NSF Example Program Data
  4                                                        :Value of N
 (-3.97,-5.04) (-4.11, 3.70) (-0.34, 1.01) ( 1.29,-0.86)
 ( 0.34,-1.50) ( 1.52,-0.43) ( 1.88,-5.38) ( 3.36, 0.65)
 ( 3.31,-3.85) ( 2.50, 3.45) ( 0.88,-1.08) ( 0.64,-1.48)
 (-1.10, 0.82) ( 1.81,-1.59) ( 3.25, 1.33) ( 1.57,-3.44)   :End of matrix A
```

9.3 Program Results

```
 F08NSF Example Program Results

 Upper Hessenberg form
                   1                  2                  3                  4
 1  (-3.9700,-5.0400) (-1.1318,-2.5693) (-4.6027,-0.1426) (-1.4249, 1.7330)
 2  (-5.4797, 0.0000) ( 1.8585,-1.5502) ( 4.4145,-0.7638) (-0.4805,-1.1976)
 3  ( 0.0000, 0.0000) ( 6.2673, 0.0000) (-0.4504,-0.0290) (-1.3467, 1.6579)
 4  ( 0.0000, 0.0000) ( 0.0000, 0.0000) (-3.5000, 0.0000) ( 2.5619,-3.3708)
```