

NAG Library Routine Document

F07VGF (DTBCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07VGF (DTBCON) estimates the condition number of a real triangular band matrix.

2 Specification

```
SUBROUTINE F07VGF (NORM, UPLO, DIAG, N, KD, AB, LDAB, RCOND, WORK, IWORK,      &
                  INFO)
INTEGER          N, KD, LDAB, IWORK(N), INFO
REAL (KIND=nag_wp) AB(LDAB,*), RCOND, WORK(3*N)
CHARACTER(1)    NORM, UPLO, DIAG
```

The routine may be called by its LAPACK name *dtbcon*.

3 Description

F07VGF (DTBCON) estimates the condition number of a real triangular band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of the condition number.

The routine computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: NORM – CHARACTER(1) *Input*
On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.
 NORM = '1' or 'O'
 $\kappa_1(A)$ is estimated.
 NORM = 'I'
 $\kappa_\infty(A)$ is estimated.
Constraint: NORM = '1', 'O' or 'I'.

- 2: UPLO – CHARACTER(1) *Input*
On entry: specifies whether A is upper or lower triangular.
 UPLO = 'U'
 A is upper triangular.
 UPLO = 'L'
 A is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 3: DIAG – CHARACTER(1) *Input*
On entry: indicates whether A is a nonunit or unit triangular matrix.
 DIAG = 'N'
 A is a nonunit triangular matrix.
 DIAG = 'U'
 A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.
Constraint: DIAG = 'N' or 'U'.
- 4: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 5: KD – INTEGER *Input*
On entry: k_d , the number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'.
Constraint: $KD \geq 0$.
- 6: AB(LDAB,*) – REAL (KIND=nag_wp) array *Input*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the n by n triangular band matrix A .
 The matrix is stored in rows 1 to $k_d + 1$, more precisely,
 if UPLO = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $AB(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;
 if UPLO = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $AB(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.
 If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.
- 7: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F07VGF (DTBCON) is called.
Constraint: $LDAB \geq KD + 1$.
- 8: RCOND – REAL (KIND=nag_wp) *Output*
On exit: an estimate of the reciprocal of the condition number of A . RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than *machine precision*, A is singular to working precision.

- 9: WORK(3 × N) – REAL (KIND=nag_wp) array Workspace
- 10: IWORK(N) – INTEGER array Workspace
- 11: INFO – INTEGER Output
- On exit:* INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07VGF (DTBCON) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2nk$ floating point operations (assuming $n \gg k$) but takes considerably longer than a call to F07VEF (DTBTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07VUF (ZTBCON).

9 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with one subdiagonal. The true condition number in the 1-norm is 69.62.

9.1 Program Text

```

Program f07vgfe

!      F07VGF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
!      Use nag_library, Only: dtbcon, nag_wp, x02ajf
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      Character (1), Parameter    :: diag = 'N', norm = '1'
!      .. Local Scalars ..
!      Real (Kind=nag_wp)          :: rcond
!      Integer                    :: i, info, j, kd, ldab, n
!      Character (1)              :: uplo

```

```

! .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: ab(:,:), work(:)
Integer, Allocatable :: iwork(:)
! .. Intrinsic Procedures ..
Intrinsic :: max, min
! .. Executable Statements ..
Write (nout,*) 'F07VGF Example Program Results'
! Skip heading in data file
Read (nin,*)
Read (nin,*) n, kd
ldab = kd + 1
Allocate (ab(ldab,n),work(3*n),iwork(n))

! Read A from data file

Read (nin,*) uplo
If (uplo=='U') Then
  Do i = 1, n
    Read (nin,*)(ab(kd+1+i-j,j),j=i,min(n,i+kd))
  End Do
Else If (uplo=='L') Then
  Do i = 1, n
    Read (nin,*)(ab(1+i-j,j),j=max(1,i-kd),i)
  End Do
End If

! Estimate condition number
! The NAG name equivalent of dtbcon is f07vgf
Call dtbcon(norm,uplo,diag,n,kd,ab,ldab,rcond,work,iwork,info)

Write (nout,*)
If (rcond>=x02ajf()) Then
  Write (nout,99999) 'Estimate of condition number =', &
    1.0E0_nag_wp/rcond
Else
  Write (nout,*) 'A is singular to working precision'
End If

99999 Format (1X,A,1P,E10.2)
End Program f07vgfe

```

9.2 Program Data

```

F07VGF Example Program Data
  4 1 :Values of N and KD
  'L' :Value of UPLO
-4.16
-2.25 4.78
      5.86 6.32
      -4.82 0.16 :End of matrix A

```

9.3 Program Results

F07VGF Example Program Results

Estimate of condition number = 6.96E+01
