NAG Library Routine Document

F07VEF (DTBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07VEF (DTBTRS) solves a real triangular band system of linear equations with multiple right-hand sides, $AX = B$ or $A^{\mathrm{T}}X = B$.

2 Specification

```
SUBROUTINE F07VEF (UPLO, TRANS, DIAG, N, KD, NRHS, AB, LDAB, B, LDB, INFO)

INTEGER            N, KD, NRHS, LDAB, LDB, INFO
REAL (KIND=nag_wp) AB(LDAB,*), B(LDB,*)
CHARACTER(1)       UPLO, TRANS, DIAG
```

The routine may be called by its LAPACK name ***dtbtrs***.

3 Description

F07VEF (DTBTRS) solves a real triangular band system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies whether A is upper or lower triangular.

UPLO = 'U'
A is upper triangular.

UPLO = 'L'
A is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: TRANS – CHARACTER(1) *Input*

On entry: indicates the form of the equations.

TRANS = 'N'
The equations are of the form $AX = B$.

TRANS = 'T' or 'C'
The equations are of the form $A^{\mathrm{T}}X = B$.

Constraint: TRANS = 'N', 'T' or 'C'.

3: DIAG – CHARACTER(1) *Input*

On entry: indicates whether A is a nonunit or unit triangular matrix.

DIAG = 'N'
A is a nonunit triangular matrix.

DIAG = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: DIAG = 'N' or 'U'.

4: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

5: KD – INTEGER *Input*

On entry: k_d, the number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'.

Constraint: KD ≥ 0.

6: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: NRHS ≥ 0.

7: AB(LDAB,$*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array AB must be at least $\max(1, \text{N})$.

On entry: the n by n triangular band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if UPLO = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in AB$(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;

if UPLO = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in AB$(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

8: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07VEF (DTBTRS) is called.

Constraint: LDAB $\geq$ KD + 1.

9: B(LDB,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

10: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07VEF (DTBTRS) is called.

Constraint: LDB $\geq \max(1, \text{N})$.

11: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $a(i,i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(k)\epsilon|A|,$$

$c(k)$ is a modest linear function of k, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A,x)\epsilon, \qquad \text{provided} \qquad c(k)\,\mathrm{cond}(A,x)\epsilon < 1,$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A,x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}\left(A^{\mathrm{T}}\right)$ to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07VHF (DTBRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07VGF (DTBCON) with NORM = 'I'.

8 Further Comments

The total number of floating point operations is approximately $2nkr$ if $k \ll n$.

The complex analogue of this routine is F07VSF (ZTBTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -16.64 & -4.16 \\ -13.78 & -16.59 \\ 13.10 & -4.94 \\ -14.14 & -9.96 \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with one subdiagonal.

9.1 Program Text

```
    Program f07vefe

!     F07VEF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dtbtrs, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
      Character (1), Parameter         :: diag = 'N', trans = 'N'
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, j, kd, ldab, ldb, n, &
                                          nrhs
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: ab(:,:), b(:,:)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: max, min
!     .. Executable Statements ..
      Write (nout,*) 'F07VEF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, kd, nrhs
      ldab = kd + 1
      ldb = n
      Allocate (ab(ldab,n),b(ldb,nrhs))

!     Read A and B from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Do i = 1, n
          Read (nin,*)(ab(kd+1+i-j,j),j=i,min(n,i+kd))
        End Do
      Else If (uplo=='L') Then
        Do i = 1, n
          Read (nin,*)(ab(1+i-j,j),j=max(1,i-kd),i)
        End Do
      End If
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Compute solution
!     The NAG name equivalent of dtbtrs is f07vef
      Call dtbtrs(uplo,trans,diag,n,kd,nrhs,ab,ldab,b,ldb,info)

!     Print solution

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)

      Else
        Write (nout,*) 'A is singular'
      End If

    End Program f07vefe
```

9.2 Program Data

```
F07VEF Example Program Data
  4  1  2                     :Values of N, KD and NRHS
  'L'                         :Value of UPLO
 -4.16
 -2.25   4.78
         5.86   6.32
               -4.82   0.16   :End of matrix A
-16.64  -4.16
-13.78 -16.59
 13.10  -4.94
-14.14  -9.96                 :End of matrix B
```

9.3 Program Results

```
 F07VEF Example Program Results

 Solution(s)
             1          2
 1      4.0000     1.0000
 2     -1.0000    -3.0000
 3      3.0000     2.0000
 4      2.0000    -2.0000
```