NAG Library Routine Document

F07TGF (DTRCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07TGF (DTRCON) estimates the condition number of a real triangular matrix.

2 Specification

```
SUBROUTINE F07TGF (NORM, UPLO, DIAG, N, A, LDA, RCOND, WORK, IWORK, INFO)

INTEGER             N, LDA, IWORK(N), INFO
REAL (KIND=nag_wp) A(LDA,*), RCOND, WORK(3*N)
CHARACTER(1)        NORM, UPLO, DIAG
```

The routine may be called by its LAPACK name ***dtrcon***.

3 Description

F07TGF (DTRCON) estimates the condition number of a real triangular matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \qquad \text{or} \qquad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1\left(A^{\mathrm{T}}\right)$.

Because the condition number is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of the condition number.

The routine computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: NORM – CHARACTER(1) *Input*

On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

NORM = '1' or 'O'
$\kappa_1(A)$ is estimated.

NORM = 'I'
$\kappa_\infty(A)$ is estimated.

Constraint: NORM = '1', 'O' or 'I'.

2: UPLO – CHARACTER(1) *Input*

On entry: specifies whether A is upper or lower triangular.

UPLO = 'U'
A is upper triangular.

UPLO = 'L'
A is lower triangular.

Constraint: UPLO = 'U' or 'L'.

3: DIAG – CHARACTER(1) *Input*

On entry: indicates whether A is a nonunit or unit triangular matrix.

DIAG = 'N'
A is a nonunit triangular matrix.

DIAG = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: DIAG = 'N' or 'U'.

4: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

5: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array A must be at least max(1, N).

On entry: the n by n triangular matrix A.

If UPLO = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced.

If UPLO = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced.

If DIAG = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

6: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07TGF (DTRCON) is called.

Constraint: LDA $\geq$ max(1, N).

7: RCOND – REAL (KIND=nag_wp) *Output*

On exit: an estimate of the reciprocal of the condition number of A. RCOND is set to zero if exact singularity is detected or if the estimate underflows. If RCOND is less than ***machine precision***, then A is singular to working precision.

8: WORK(3 $\times$ N) – REAL (KIND=nag_wp) array *Workspace*

9: IWORK(N) – INTEGER array *Workspace*

10: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07TGF (DTRCON) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{T}}x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately n^2 floating point operations but takes considerably longer than a call to F07TEF (DTRTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07TUF (ZTRCON).

9 Example

This example estimates the condition number in the 1-norm of the matrix A, where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix}.$$

The true condition number in the 1-norm is 116.41.

9.1 Program Text

```
    Program f07tgfe

!     F07TGF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dtrcon, nag_wp, x02ajf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
      Character (1), Parameter         :: diag = 'N', norm = '1'
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: rcond
      Integer                          :: i, info, lda, n
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:), work(:)
      Integer, Allocatable             :: iwork(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07TGF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      Allocate (a(lda,n),work(3*n),iwork(n))

!     Read A from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)(a(i,i:n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)(a(i,1:i),i=1,n)
      End If

!     Estimate condition number
!     The NAG name equivalent of dtrcon is f07tgf
```

```
      Call dtrcon(norm,uplo,diag,n,a,lda,rcond,work,iwork,info)

      Write (nout,*)
      If (rcond>=x02ajf()) Then
        Write (nout,99999) 'Estimate of condition number =', &
          1.0E0_nag_wp/rcond
      Else
        Write (nout,*) 'A is singular to working precision'
      End If

99999 Format (1X,A,1P,E10.2)
    End Program f07tgfe
```

9.2 Program Data

```
F07TGF Example Program Data
  4                          :Value of N
  'L'                        :Value of UPLO
  4.30
 -3.96  -4.87
  0.40   0.31  -8.02
 -0.27   0.07  -5.95   0.12   :End of matrix A
```

9.3 Program Results

```
 F07TGF Example Program Results

 Estimate of condition number =  1.16E+02
```