

NAG Library Routine Document

F07QUF (ZSPCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07QUF (ZSPCON) estimates the condition number of a complex symmetric matrix A , where A has been factorized by F07QRF (ZSPTRF), using packed storage.

2 Specification

SUBROUTINE F07QUF (UPLO, N, AP, IPIV, ANORM, RCOND, WORK, INFO)

INTEGER N, IPIV(*), INFO
 REAL (KIND=nag_wp) ANORM, RCOND
 COMPLEX (KIND=nag_wp) AP(*), WORK(2*N)
 CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zspcon*.

3 Description

F07QUF (ZSPCON) estimates the condition number (in the 1-norm) of a complex symmetric matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The routine should be preceded by a call to F06UGF to compute $\|A\|_1$ and a call to F07QRF (ZSPTRF) to compute the Bunch–Kaufman factorization of A . The routine then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'

$A = PUDU^T P^T$, where U is upper triangular.

UPLO = 'L'

$A = PLDL^T P^T$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: AP(*) – COMPLEX (KIND=nag_wp) array *Input*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the factorization of A stored in packed form, as returned by F07QRF (ZSPTRF).
- 4: IPIV(*) – INTEGER array *Input*
Note: the dimension of the array IPIV must be at least $\max(1, N)$.
On entry: details of the interchanges and the block structure of D , as returned by F07QRF (ZSPTRF).
- 5: ANORM – REAL (KIND=nag_wp) *Input*
On entry: the 1-norm of the **original** matrix A , which may be computed by calling F06UGF with its parameter NORM = '1'. ANORM must be computed either **before** calling F07QRF (ZSPTRF) or else from a **copy** of the original matrix A .
Constraint: ANORM ≥ 0.0 .
- 6: RCOND – REAL (KIND=nag_wp) *Output*
On exit: an estimate of the reciprocal of the condition number of A . RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than **machine precision**, A is singular to working precision.
- 7: WORK($2 \times N$) – COMPLEX (KIND=nag_wp) array *Workspace*
- 8: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07QUF (ZSPCON) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating point operations but takes considerably longer than a call to F07QSF (ZSPTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this routine is F07PGF (DSPCON).

9 Example

This example estimates the condition number in the 1-norm (or ∞ -norm) of the matrix A , where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}.$$

Here A is symmetric, stored in packed form, and must first be factorized by F07QRF (ZSPTRF). The true condition number in the 1-norm is 32.92.

9.1 Program Text

```

Program f07qufe

!       F07QUF Example Program Text

!       Mark 24 Release. NAG Copyright 2012.

!       .. Use Statements ..
Use nag_library, Only: nag_wp, x02ajf, zlansp => f06ugf, zspcon, zsptrf
!       .. Implicit None Statement ..
Implicit None
!       .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!       .. Local Scalars ..
Real (Kind=nag_wp)         :: anorm, rcond
Integer                    :: i, info, j, n
Character (1)              :: uplo
!       .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: ap(:), work(:)
Real (Kind=nag_wp), Allocatable  :: rwork(:)
Integer, Allocatable        :: ipiv(:)
!       .. Executable Statements ..
Write (nout,*) 'F07QUF Example Program Results'
!       Skip heading in data file
Read (nin,*)
Read (nin,*) n

Allocate (ap(n*(n+1)/2),work(2*n),rwork(n),ipiv(n))

!       Read A from data file

Read (nin,*) uplo
If (uplo=='U') Then
  Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
Else If (uplo=='L') Then
  Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
End If

!       Compute norm of A
!       f06ugf is the NAG name equivalent of the LAPACK auxiliary zlansp
anorm = zlansp('1-norm',uplo,n,ap,rwork)

!       Factorize A
!       The NAG name equivalent of zsptrf is f07qrf
Call zsptrf(uplo,n,ap,ipiv,info)

Write (nout,*)
If (info==0) Then

!       Estimate condition number
!       The NAG name equivalent of zspcon is f07quf
Call zspcon(uplo,n,ap,ipiv,anorm,rcond,work,info)

If (rcond>=x02ajf()) Then
  Write (nout,99999) 'Estimate of condition number =', &
    1.0E0_nag_wp/rcond

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      Else
        Write (nout,*) 'A is singular to working precision'
      End If
    Else
      Write (nout,*) 'The factor D is singular'
    End If

99999 Format (1X,A,1P,E10.2)
      End Program f07qufe
```

9.2 Program Data

F07QUF Example Program Data

```
  4                                     :Value of N
  'L'                                   :Value of UPLO
(-0.39,-0.71)
( 5.14,-0.64) ( 8.86, 1.81)
(-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)
( 3.80, 0.92) ( 5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12) :End of matrix A
```

9.3 Program Results

F07QUF Example Program Results

Estimate of condition number = 2.06E+01
