NAG Library Routine Document

F07PJF (DSPTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07PJF (DSPTRI) computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by F07PDF (DSPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07PJF (UPLO, N, AP, IPIV, WORK, INFO)

INTEGER            N, IPIV(*), INFO
REAL (KIND=nag_wp) AP(*), WORK(N)
CHARACTER(1)       UPLO
```

The routine may be called by its LAPACK name ***dsptri***.

3 Description

F07PJF (DSPTRI) is used to compute the inverse of a real symmetric indefinite matrix A, the routine must be preceded by a call to F07PDF (DSPTRF), which computes the Bunch–Kaufman factorization of A, using packed storage.

If UPLO = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{T}}P^{\mathrm{T}}XPU = D^{-1}$.

If UPLO = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{T}}P^{\mathrm{T}}XPL = D^{-1}$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'
$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: AP($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array AP must be at least $\max(1, \mathrm{N} \times (\mathrm{N}+1)/2)$.

On entry: the factorization of A stored in packed form, as returned by F07PDF (DSPTRF).

On exit: the factorization is overwritten by the n by n matrix A^{-1}.

More precisely,

if UPLO = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in $\mathrm{AP}(i+j(j-1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in $\mathrm{AP}(i+(2n-j)(j-1)/2)$ for $i \geq j$.

4: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On entry: details of the interchanges and the block structure of D, as returned by F07PDF (DSPTRF).

5: WORK(N) – REAL (KIND=nag_wp) array *Workspace*

6: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $d(i,i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $\left|DU^{\mathrm{T}}P^{\mathrm{T}}XPU - I\right| \leq c(n)\epsilon\left(|D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;

if UPLO = 'L', $\left|DL^{\mathrm{T}}P^{\mathrm{T}}XPL - I\right| \leq c(n)\epsilon\left(|D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this routine are F07PWF (ZHPTRI) for Hermitian matrices and F07QWF (ZSPTRI) for symmetric matrices.

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite, stored in packed form, and must first be factorized by F07PDF (DSPTRF).

9.1 Program Text

```
    Program f07pjfe

!     F07PJF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dsptrf, dsptri, nag_wp, x04ccf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, j, n
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: ap(:), work(:)
      Integer, Allocatable             :: ipiv(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07PJF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (ap(n*(n+1)/2),work(n),ipiv(n))

!     Read A from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
      End If

!     Factorize A
!     The NAG name equivalent of dsptrf is f07pdf
      Call dsptrf(uplo,n,ap,ipiv,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute inverse of A
!       The NAG name equivalent of dsptri is f07pjf
        Call dsptri(uplo,n,ap,ipiv,work,info)

!       Print inverse

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04ccf(uplo,'Nonunit',n,ap,'Inverse',ifail)
```

```
      Else
        Write (nout,*) 'The factor D is singular'
      End If

    End Program f07pjfe
```

9.2 Program Data

```
F07PJF Example Program Data
  4                           :Value of N
  'L'                         :Value of UPLO
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

9.3 Program Results

```
 F07PJF Example Program Results

 Inverse
            1          2          3          4
 1      0.7485
 2      0.5221    -0.1605
 3     -1.0058    -0.3131     1.3501
 4     -1.4386    -0.7440     2.0667     2.4547
```