

# NAG Library Routine Document

## F07NNF (ZSYSV)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07NNF (ZSYSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where  $A$  is an  $n$  by  $n$  symmetric matrix and  $X$  and  $B$  are  $n$  by  $r$  matrices.

### 2 Specification

SUBROUTINE F07NNF (UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK, INFO)

INTEGER N, NRHS, LDA, IPIV(\*), LDB, LWORK, INFO  
 COMPLEX (KIND=nag\_wp) A(LDA,\*), B(LDB,\*), WORK(max(1,LWORK))  
 CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zsysv*.

### 3 Description

F07NNF (ZSYSV) uses the diagonal pivoting method to factor  $A$  as  $A = UDU^T$  if UPLO = 'U' or  $A = LDL^T$  if UPLO = 'L', where  $U$  (or  $L$ ) is a product of permutation and unit upper (lower) triangular matrices, and  $D$  is symmetric and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of  $A$  is then used to solve the system of equations  $AX = B$ .

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

### 5 Parameters

1: UPLO – CHARACTER(1) *Input*

*On entry:* if UPLO = 'U', the upper triangle of  $A$  is stored.

If UPLO = 'L', the lower triangle of  $A$  is stored.

*Constraint:* UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

*On entry:*  $n$ , the number of linear equations, i.e., the order of the matrix  $A$ .

*Constraint:*  $N \geq 0$ .

- 3: NRHS – INTEGER *Input*  
*On entry:*  $r$ , the number of right-hand sides, i.e., the number of columns of the matrix  $B$ .  
*Constraint:*  $\text{NRHS} \geq 0$ .
- 4: A(LDA,\*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array  $A$  must be at least  $\max(1, N)$ .  
*On entry:* the  $n$  by  $n$  symmetric matrix  $A$ .  
 If UPLO = 'U', the upper triangular part of  $A$  must be stored and the elements of the array below the diagonal are not referenced.  
 If UPLO = 'L', the lower triangular part of  $A$  must be stored and the elements of the array above the diagonal are not referenced.  
*On exit:* if INFO = 0, the block diagonal matrix  $D$  and the multipliers used to obtain the factor  $U$  or  $L$  from the factorization  $A = UDU^T$  or  $A = LDL^T$  as computed by F07NRF (ZSYTRF).
- 5: LDA – INTEGER *Input*  
*On entry:* the first dimension of the array  $A$  as declared in the (sub)program from which F07NNF (ZSYSV) is called.  
*Constraint:*  $\text{LDA} \geq \max(1, N)$ .
- 6: IPIV(\*) – INTEGER array *Output*  
**Note:** the dimension of the array IPIV must be at least  $\max(1, N)$ .  
*On exit:* details of the interchanges and the block structure of  $D$ . More precisely,  
 if  $\text{IPIV}(i) = k > 0$ ,  $d_{ii}$  is a 1 by 1 pivot block and the  $i$ th row and column of  $A$  were interchanged with the  $k$ th row and column;  
 if UPLO = 'U' and  $\text{IPIV}(i-1) = \text{IPIV}(i) = -l < 0$ ,  $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$  is a 2 by 2 pivot block and the  $(i-1)$ th row and column of  $A$  were interchanged with the  $l$ th row and column;  
 if UPLO = 'L' and  $\text{IPIV}(i) = \text{IPIV}(i+1) = -m < 0$ ,  $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$  is a 2 by 2 pivot block and the  $(i+1)$ th row and column of  $A$  were interchanged with the  $m$ th row and column.
- 7: B(LDB,\*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array  $B$  must be at least  $\max(1, \text{NRHS})$ .  
**Note:** To solve the equations  $Ax = b$ , where  $b$  is a single right-hand side,  $B$  may be supplied as a one-dimensional array with length  $\text{LDB} = \max(1, N)$ .  
*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .  
*On exit:* if INFO = 0, the  $n$  by  $r$  solution matrix  $X$ .
- 8: LDB – INTEGER *Input*  
*On entry:* the first dimension of the array  $B$  as declared in the (sub)program from which F07NNF (ZSYSV) is called.  
*Constraint:*  $\text{LDB} \geq \max(1, N)$ .
- 9: WORK(max(1, LWORK)) – COMPLEX (KIND=nag\_wp) array *Workspace*  
*On exit:* if INFO = 0, WORK(1) returns the optimal LWORK.

10: LWORK – INTEGER

*Input*

*On entry:* the dimension of the array WORK as declared in the (sub)program from which F07NNF (ZSYSV) is called.

$LWORK \geq 1$ , and for best performance  $LWORK \geq \max(1, N \times nb)$ , where  $nb$  is the optimal block size for F07NRF (ZSYTRF).

If  $LWORK = -1$ , a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued.

11: INFO – INTEGER

*Output*

*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO &lt; 0

If  $INFO = -i$ , the  $i$ th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO &gt; 0

If  $INFO = i$ ,  $d_{ii}$  is exactly zero. The factorization has been completed, but the block diagonal matrix  $D$  is exactly singular, so the solution could not be computed.

## 7 Accuracy

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where  $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$ , the condition number of  $A$  with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) and Chapter 11 of Higham (2002) for further details.

F07NPF (ZSYSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04DHF solves  $Ax = b$  and returns a forward error bound and condition estimate. F04DHF calls F07NNF (ZSYSV) to solve the equations.

## 8 Further Comments

The total number of floating point operations is approximately  $\frac{4}{3}n^3 + 8n^2r$ , where  $r$  is the number of right-hand sides.

The real analogue of this routine is F07MAF (DSYSV). The complex Hermitian analogue of this routine is F07MNF (ZHESV).

## 9 Example

This example solves the equations

$$Ax = b,$$

where  $A$  is the complex symmetric matrix

$$A = \begin{pmatrix} -0.56 + 0.12i & -1.54 - 2.86i & 5.32 - 1.59i & 3.80 + 0.92i \\ -1.54 - 2.86i & -2.83 - 0.03i & -3.52 + 0.58i & -7.86 - 2.96i \\ 5.32 - 1.59i & -3.52 + 0.58i & 8.86 + 1.81i & 5.14 - 0.64i \\ 3.80 + 0.92i & -7.86 - 2.96i & 5.14 - 0.64i & -0.39 - 0.71i \end{pmatrix}$$

and

$$b = \begin{pmatrix} -6.43 + 19.24i \\ -0.49 - 1.47i \\ -48.18 + 66.00i \\ -55.64 + 41.22i \end{pmatrix}.$$

Details of the factorization of  $A$  are also output.

### 9.1 Program Text

```

Program f07nnfe

!      F07NNF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
Use nag_library, Only: nag_wp, x04dbf, zsysv
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nb = 64, nin = 5, nout = 6
!      .. Local Scalars ..
Integer                     :: i, ifail, info, lda, lwork, n
!      .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: a(:,,:), b(:), work(:)
Integer, Allocatable         :: ipiv(:)
Character (1)                :: clabs(1), rlabs(1)
!      .. Executable Statements ..
Write (nout,*) 'F07NNF Example Program Results'
Write (nout,*)
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n
lda = n
lwork = nb*n
Allocate (a(lda,n),b(n),work(lwork),ipiv(n))

!      Read the upper triangular part of the matrix A from data file

Read (nin,*)(a(i,i:n),i=1,n)

!      Read b from data file

Read (nin,*) b(1:n)

!      Solve the equations Ax = b for x

!      The NAG name equivalent of zsysv is f07nnf
Call zsysv('Upper',n,1,a,lda,ipiv,b,n,work,lwork,info)

If (info==0) Then

!      Print solution

```

```

      Write (nout,*) 'Solution'
      Write (nout,99999) b(1:n)

!      Print details of factorization

      Write (nout,*)
      Flush (nout)

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04dbf('Upper', 'Non-unit diagonal', n, n, a, lda, 'Bracketed', 'F7.4', &
        'Details of the factorization', 'Integer', rlabs, 'Integer', clabs, 80, 0, &
        ifail)

!      Print pivot indices

      Write (nout,*)
      Write (nout,*) 'Pivot indices'
      Write (nout,99998) ipiv(1:n)

      Else
        Write (nout,99997) 'The diagonal block ', info, ' of D is zero'
      End If

99999 Format ((4X,4('(',F7.4,',',F7.4,') ')))
99998 Format (1X,7I11)
99997 Format (1X,A,I3,A)
      End Program f07nnfe

```

## 9.2 Program Data

F07NNF Example Program Data

```

      4                                     :Value of N
      (-0.56, 0.12) (-1.54, -2.86) ( 5.32, -1.59) ( 3.80, 0.92)
                ( -2.83, -0.03) (-3.52, 0.58) (-7.86, -2.96)
                        ( 8.86, 1.81) ( 5.14, -0.64)
                                (-0.39, -0.71) :End matrix A
      (-6.43, 19.24) (-0.49, -1.47) (-48.18, 66.00) (-55.64, 41.22) :End vector b

```

## 9.3 Program Results

F07NNF Example Program Results

```

Solution
(-4.0000, 3.0000) ( 3.0000,-2.0000) (-2.0000, 5.0000) ( 1.0000,-1.0000)

```

Details of the factorization

```

      1           2           3           4
1 (-2.0954,-2.2011) (-0.1071,-0.3157) (-0.4823, 0.0150) ( 0.4426, 0.1936)
2 ( 4.4079, 5.3991) (-0.6078, 0.2811) ( 0.5279,-0.3715)
3 (-2.8300,-0.0300) (-7.8600,-2.9600)
4 (-0.3900,-0.7100)

```

Pivot indices

```

      1           2           -2           -2

```

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