NAG Library Routine Document

F07MJF (DSYTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07MJF (DSYTRI) computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by F07MDF (DSYTRF).

2 Specification

```
SUBROUTINE F07MJF (UPLO, N, A, LDA, IPIV, WORK, INFO)

INTEGER            N, LDA, IPIV(*), INFO
REAL (KIND=nag_wp) A(LDA,*), WORK(N)
CHARACTER(1)       UPLO
```

The routine may be called by its LAPACK name ***dsytri***.

3 Description

F07MJF (DSYTRI) is used to compute the inverse of a real symmetric indefinite matrix A, the routine must be preceded by a call to F07MDF (DSYTRF), which computes the Bunch–Kaufman factorization of A.

If UPLO = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{T}}P^{\mathrm{T}}XPU = D^{-1}$ for X.

If UPLO = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{T}}P^{\mathrm{T}}XPL = D^{-1}$ for X.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'
$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least max(1, N).

On entry: details of the factorization of A, as returned by F07MDF (DSYTRF).

On exit: the factorization is overwritten by the n by n symmetric matrix A^{-1}.

If UPLO = 'U', the upper triangle of A^{-1} is stored in the upper triangular part of the array.

If UPLO = 'L', the lower triangle of A^{-1} is stored in the lower triangular part of the array.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07MJF (DSYTRI) is called.

Constraint: LDA $\geq$ max(1, N).

5: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least max(1, N).

On entry: details of the interchanges and the block structure of D, as returned by F07MDF (DSYTRF).

6: WORK(N) – REAL (KIND=nag_wp) array *Workspace*

7: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $d(i,i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $\left|DU^{\mathrm{T}}P^{\mathrm{T}}XPU - I\right| \leq c(n)\epsilon\left(|D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;

if UPLO = 'L', $\left|DL^{\mathrm{T}}P^{\mathrm{T}}XPL - I\right| \leq c(n)\epsilon\left(|D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this routine are F07MWF (ZHETRI) for Hermitian matrices and F07NWF (ZSYTRI) for symmetric matrices.

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite and must first be factorized by F07MDF (DSYTRF).

9.1 Program Text

```
    Program f07mjfe

!     F07MJF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dsytrf, dsytri, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, lwork, n
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:), work(:)
      Integer, Allocatable             :: ipiv(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07MJF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      lwork = 64*n
      Allocate (a(lda,n),work(lwork),ipiv(n))

!     Read A from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)(a(i,i:n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)(a(i,1:i),i=1,n)
      End If

!     Factorize A
!     The NAG name equivalent of dsytrf is f07mdf
      Call dsytrf(uplo,n,a,lda,ipiv,work,lwork,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute inverse of A
!       The NAG name equivalent of dsytri is f07mjf
        Call dsytri(uplo,n,a,lda,ipiv,work,info)

!       Print inverse

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf(uplo,'Nonunit',n,n,a,lda,'Inverse',ifail)
```

```
      Else
        Write (nout,*) 'The factor D is singular'
      End If

    End Program f07mjfe
```

9.2 Program Data

```
F07MJF Example Program Data
  4                           :Value of N
  'L'                         :Value of UPLO
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

9.3 Program Results

```
 F07MJF Example Program Results

 Inverse
            1          2          3          4
 1      0.7485
 2      0.5221    -0.1605
 3     -1.0058    -0.3131     1.3501
 4     -1.4386    -0.7440     2.0667     2.4547
```