NAG Library Routine Document

F07JEF (DPTTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JEF (DPTTRS) computes the solution to a real system of linear equations $AX = B$, where A is an n by n symmetric positive definite tridiagonal matrix and X and B are n by r matrices, using the LDL^{T} factorization returned by F07JDF (DPTTRF).

2 Specification

```
SUBROUTINE F07JEF (N, NRHS, D, E, B, LDB, INFO)

INTEGER            N, NRHS, LDB, INFO
REAL (KIND=nag_wp) D(*), E(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***dpttrs***.

3 Description

F07JEF (DPTTRS) should be preceded by a call to F07JDF (DPTTRF), which computes a modified Cholesky factorization of the matrix A as

$$A = LDL^{\mathrm{T}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix, with positive diagonal elements. F07JEF (DPTTRS) then utilizes the factorization to solve the required equations. Note that the factorization may also be regarded as having the form $U^{\mathrm{T}}DU$, where U is a unit upper bidiagonal matrix.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

5 Parameters

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: NRHS ≥ 0.

3: D($*$) – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

4: E$(*)$ – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array E must be at least $\max(1, \mathrm{N} - 1)$.

On entry: must contain the $(n - 1)$ subdiagonal elements of the unit lower bidiagonal matrix L. (E can also be regarded as the superdiagonal of the unit upper bidiagonal matrix U from the $U^{\mathrm{T}}DU$ factorization of A.)

5: B(LDB,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r matrix of right-hand sides B.

On exit: the n by r solution matrix X.

6: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07JEF (DPTTRS) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

7: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of this routine F07JGF (DPTCON) can be used to estimate the condition number of A and F07JHF (DPTRFS) can be used to obtain approximate error bounds.

8 Further Comments

The total number of floating point operations required to solve the equations $AX = B$ is proportional to nr.

The complex analogue of this routine is F07JSF (ZPTTRS).

9 Example

This example solves the equations

$$AX = B,$$

where A is the symmetric positive definite tridiagonal matrix

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 6.0 & 10.0 \\ 9.0 & 4.0 \\ 2.0 & 9.0 \\ 14.0 & 65.0 \\ 7.0 & 23.0 \end{pmatrix}.$$

9.1 Program Text

```
    Program f07jefe

!     F07JEF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dpttrf, dpttrs, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, ldb, n, nrhs
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: b(:,:), d(:), e(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07JEF Example Program Results'
      Write (nout,*)
      Flush (nout)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs
      ldb = n
      Allocate (b(ldb,nrhs),d(n),e(n-1))

!     Read the upper bidiagonal part of the tridiagonal matrix A from
!     data file

      Read (nin,*) e(1:n-1)
      Read (nin,*) d(1:n)

!     Read the right hand matrix B

      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Factorize the tridiagonal matrix A
!     The NAG name equivalent of dpttrf is f07jdf
      Call dpttrf(n,d,e,info)

      If (info==0) Then

!       Solve the equations AX = B
!       The NAG name equivalent of dpttrs is f07jef
        Call dpttrs(n,nrhs,d,e,b,ldb,info)

!       Print the solution

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)
```

```
      Else
        Write (nout,99999) 'The leading minor of order ', info, &
          ' is not positive definite'
      End If

99999 Format (1X,A,I3,A)
    End Program f07jefe
```

9.2 Program Data

```
F07JEF Example Program Data
  5     2                       :Values of N and NRHS
       -2.0  -6.0  15.0   8.0 :End of super-diagonal E
  4.0  10.0  29.0  25.0   5.0 :End of diagonal D
  6.0  10.0
  9.0   4.0
  2.0   9.0
 14.0  65.0
  7.0  23.0                     :End of matrix B
```

9.3 Program Results

```
 F07JEF Example Program Results

 Solution(s)
              1          2
 1       2.5000     2.0000
 2       2.0000    -1.0000
 3       1.0000    -3.0000
 4      -1.0000     6.0000
 5       3.0000    -5.0000
```