NAG Library Routine Document

F07HSF (ZPBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07HSF (ZPBTRS) solves a complex Hermitian positive definite band system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07HRF (ZPBTRF).

2 Specification

```
SUBROUTINE F07HSF (UPLO, N, KD, NRHS, AB, LDAB, B, LDB, INFO)

INTEGER                 N, KD, NRHS, LDAB, LDB, INFO
COMPLEX (KIND=nag_wp)   AB(LDAB,*), B(LDB,*)
CHARACTER(1)            UPLO
```

The routine may be called by its LAPACK name ***zpbtrs***.

3 Description

F07HSF (ZPBTRS) is used to solve a complex Hermitian positive definite band system of linear equations $AX = B$, the routine must be preceded by a call to F07HRF (ZPBTRF) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If UPLO = 'U', $A = U^{\mathrm{H}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{H}}Y = B$ and then $UX = Y$.

If UPLO = 'L', $A = LL^{\mathrm{H}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{H}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'
: $A = U^{\mathrm{H}}U$, where U is upper triangular.

UPLO = 'L'
: $A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $N \geq 0$.

3: KD – INTEGER *Input*

On entry: k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: $KD \geq 0$.

4: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: $NRHS \geq 0$.

5: AB(LDAB,$*$) – COMPLEX (KIND=nag_wp) array *Input*

Note: the second dimension of the array AB must be at least $\max(1, N)$.

On entry: the Cholesky factor of A, as returned by F07HRF (ZPBTRF).

6: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07HSF (ZPBTRS) is called.

Constraint: $LDAB \geq KD + 1$.

7: B(LDB,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07HSF (ZPBTRS) is called.

Constraint: $LDB \geq \max(1, N)$.

9: INFO – INTEGER *Output*

On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If $INFO = -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if UPLO = 'U', $|E| \leq c(k+1)\epsilon|U^{H}||U|$;

if UPLO = 'L', $|E| \leq c(k+1)\epsilon|L||L^{H}|$,

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k+1)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07HVF (ZPBRFS), and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling F07HUF (ZPBCON).

8 Further Comments

The total number of real floating point operations is approximately $16nkr$, assuming $n \gg k$.

This routine may be followed by a call to F07HVF (ZPBRFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07HEF (DPBTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 9.39+0.00i & 1.08-1.73i & 0.00+0.00i & 0.00+0.00i \\ 1.08+1.73i & 1.69+0.00i & -0.04+0.29i & 0.00+0.00i \\ 0.00+0.00i & -0.04-0.29i & 2.65+0.00i & -0.33+2.24i \\ 0.00+0.00i & 0.00+0.00i & -0.33-2.24i & 2.17+0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -12.42+68.42i & 54.30-56.56i \\ -9.93+\ \ 0.88i & 18.32+\ \ 4.76i \\ -27.30-\ \ 0.01i & -4.40+\ \ 9.97i \\ 5.31+23.63i & 9.43+\ \ 1.41i \end{pmatrix}.$$

Here A is Hermitian positive definite, and is treated as a band matrix, which must first be factorized by F07HRF (ZPBTRF).

9.1 Program Text

```
    Program f07hsfe

!     F07HSF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zpbtrf, zpbtrs
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, j, kd, ldab, ldb, n, &
                                          nrhs
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: ab(:,:), b(:,:)
      Character (1)                    :: clabs(1), rlabs(1)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: max, min
!     .. Executable Statements ..
      Write (nout,*) 'F07HSF Example Program Results'
```

```
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, kd, nrhs
      ldab = kd + 1
      ldb = n
      Allocate (ab(ldab,n),b(ldb,nrhs))

!     Read A and B from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Do i = 1, n
          Read (nin,*)(ab(kd+1+i-j,j),j=i,min(n,i+kd))
        End Do
      Else If (uplo=='L') Then
        Do i = 1, n
          Read (nin,*)(ab(1+i-j,j),j=max(1,i-kd),i)
        End Do
      End If
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Factorize A
!     The NAG name equivalent of zpbtrf is f07hrf
      Call zpbtrf(uplo,n,kd,ab,ldab,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute solution
!       The NAG name equivalent of zpbtrs is f07hsf
        Call zpbtrs(uplo,n,kd,nrhs,ab,ldab,b,ldb,info)

!       Print solution

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04dbf('General',' ',n,nrhs,b,ldb,'Bracketed','F7.4', &
          'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

      Else
        Write (nout,*) 'A is not positive definite'
      End If

    End Program f07hsfe
```

9.2 Program Data

```
F07HSF Example Program Data
  4  1  2                                          :Values of N, KD and NRHS
  'L'                                                  :Value of UPLO
 ( 9.39, 0.00)
 ( 1.08, 1.73) ( 1.69, 0.00)
               (-0.04,-0.29) ( 2.65, 0.00)
                             (-0.33,-2.24) ( 2.17, 0.00)  :End of matrix A
 (-12.42,68.42) (54.30,-56.56)
 ( -9.93, 0.88) (18.32,  4.76)
 (-27.30,-0.01) (-4.40,  9.97)
 (  5.31,23.63) ( 9.43,  1.41)                         :End of matrix B
```

9.3 Program Results

```
F07HSF Example Program Results

Solution(s)
                   1                  2
1  (-1.0000, 8.0000) ( 5.0000,-6.0000)
2  ( 2.0000,-3.0000) ( 2.0000, 3.0000)
3  (-4.0000,-5.0000) (-8.0000, 4.0000)
4  ( 7.0000, 6.0000) (-1.0000,-7.0000)
```