NAG Library Routine Document

F07FJF (DPOTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07FJF (DPOTRI) computes the inverse of a real symmetric positive definite matrix A, where A has been factorized by F07FDF (DPOTRF).

2 Specification

```
SUBROUTINE F07FJF (UPLO, N, A, LDA, INFO)

INTEGER            N, LDA, INFO
REAL (KIND=nag_wp) A(LDA,*)
CHARACTER(1)       UPLO
```

The routine may be called by its LAPACK name ***dpotri***.

3 Description

F07FJF (DPOTRI) is used to compute the inverse of a real symmetric positive definite matrix A, the routine must be preceded by a call to F07FDF (DPOTRF), which computes the Cholesky factorization of A.

If UPLO = 'U', $A = U^{\mathrm{T}}U$ and A^{-1} is computed by first inverting U and then forming $\left(U^{-1}\right)U^{-\mathrm{T}}$.

If UPLO = 'L', $A = LL^{\mathrm{T}}$ and A^{-1} is computed by first inverting L and then forming $L^{-\mathrm{T}}\left(L^{-1}\right)$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'
$A = U^{\mathrm{T}}U$, where U is upper triangular.

UPLO = 'L'
$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the upper triangular matrix U if UPLO = 'U' or the lower triangular matrix L if UPLO = 'L', as returned by F07FDF (DPOTRF).

On exit: U is overwritten by the upper triangle of A^{-1} if UPLO = 'U'; L is overwritten by the lower triangle of A^{-1} if UPLO = 'L'.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07FJF (DPOTRI) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the ith diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \qquad \text{and} \qquad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n, ϵ is the ***machine precision*** and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2\|A^{-1}\|_2.$$

8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}n^3$.

The complex analogue of this routine is F07FWF (ZPOTRI).

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive definite and must first be factorized by F07FDF (DPOTRF).

9.1 Program Text

```
    Program f07fjfe

!     F07FJF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dpotrf, dpotri, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, n
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:)
!     .. Executable Statements ..
      Write (nout,*) 'F07FJF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      Allocate (a(lda,n))

!     Read A from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)(a(i,i:n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)(a(i,1:i),i=1,n)
      End If

!     Factorize A
!     The NAG name equivalent of dpotrf is f07fdf
      Call dpotrf(uplo,n,a,lda,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute inverse of A
!       The NAG name equivalent of dpotri is f07fjf
        Call dpotri(uplo,n,a,lda,info)

!       Print inverse

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf(uplo,'Nonunit',n,n,a,lda,'Inverse',ifail)

      Else
        Write (nout,*) 'A is not positive definite'
      End If

    End Program f07fjfe
```

9.2 Program Data

```
F07FJF Example Program Data
  4                          :Value of N
  'L'                        :Value of UPLO
  4.16
 -3.12   5.03
  0.56  -0.83   0.76
 -0.10   1.18   0.34   1.18   :End of matrix A
```

9.3 Program Results

```
 F07FJF Example Program Results

 Inverse
              1          2          3          4
 1       0.6995
 2       0.7769     1.4239
 3       0.7508     1.8255     4.0688
 4      -0.9340    -1.8841    -2.9342     3.4978
```