NAG Library Routine Document

F07FFF (DPOEQU)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07FFF (DPOEQU) computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive definite matrix A and reduce its condition number.

2 Specification

```
SUBROUTINE F07FFF (N, A, LDA, S, SCOND, AMAX, INFO)

INTEGER              N, LDA, INFO
REAL (KIND=nag_wp) A(LDA,*), S(N), SCOND, AMAX
```

The routine may be called by its LAPACK name ***dpoequ***.

3 Description

F07FFF (DPOEQU) computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B, $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham (2002)).

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the matrix A whose scaling factors are to be computed. Only the diagonal elements of the array A are referenced.

3: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07FFF (DPOEQU) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

4: S(N) – REAL (KIND=nag_wp) array *Output*

On exit: if INFO $= 0$, S contains the diagonal elements of the scaling matrix S.

5: SCOND – REAL (KIND=nag_wp) *Output*

On exit: if INFO $= 0$, SCOND contains the ratio of the smallest value of S to the largest value of S. If SCOND ≥ 0.1 and AMAX is neither too large nor too small, it is not worth scaling by S.

6: AMAX – REAL (KIND=nag_wp) *Output*

On exit: $\max \lvert a_{ij} \rvert$. If AMAX is very close to overflow or underflow, the matrix A should be scaled.

7: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the ith diagonal element of A is not positive (and hence A cannot be positive definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Further Comments

The complex analogue of this routine is F07FTF (ZPOEQU).

9 Example

This example equilibrates the symmetric positive definite matrix A given by

$$A = \begin{pmatrix} 4.16 & -3.12 \times 10^5 & 0.56 & -0.10 \\ -3.12 \times 10^5 & 5.03 \times 10^{10} & -0.83 \times 10^5 & 1.18 \times 10^5 \\ 0.56 & -0.83 \times 10^5 & 0.76 & 0.34 \\ -0.10 & 1.18 \times 10^5 & 0.34 & 1.18 \end{pmatrix}.$$

Details of the scaling factors and the scaled matrix are output.

9.1 Program Text

```
    Program f07fffe

!     F07FFF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dpoequ, dscal, f06fcf, nag_wp, x02ajf, x02amf,    &
                             x02bhf, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
```

```
      Real (Kind=nag_wp), Parameter    :: one = 1.0_nag_wp
      Real (Kind=nag_wp), Parameter    :: thresh = 0.1_nag_wp
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: amax, big, scond, small
      Integer                          :: i, ifail, info, j, lda, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:), s(:)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: real
!     .. Executable Statements ..
      Write (nout,*) 'F07FFF Example Program Results'
      Write (nout,*)
      Flush (nout)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      Allocate (a(lda,n),s(n))

!     Read the upper triangular part of the matrix A from data file

      Read (nin,*)(a(i,i:n),i=1,n)

!     Print the matrix A

!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04caf('Upper','Non-unit',n,n,a,lda,'Matrix A',ifail)

      Write (nout,*)

!     Compute diagonal scaling factors
!     The NAG name equivalent of dpoequ is f07fff
      Call dpoequ(n,a,lda,s,scond,amax,info)

      If (info>0) Then
        Write (nout,99999) 'Diagonal element', info, ' of A is non positive'
      Else

!       Print SCOND, AMAX and the scale factors

        Write (nout,99998) 'SCOND =', scond, ', AMAX =', amax
        Write (nout,*)
        Write (nout,*) 'Diagonal scaling factors'
        Write (nout,99997) s(1:n)
        Write (nout,*)
        Flush (nout)

!       Compute values close to underflow and overflow

        small = x02amf()/(x02ajf()*real(x02bhf(),kind=nag_wp))
        big = one/small
        If ((scond<thresh) .Or. (amax<small) .Or. (amax>big)) Then

!         Scale A
!         The NAG name equivalent of dscal is f06edf
          Do j = 1, n
            Call dscal(j,s(j),a(1,j),1)
            Call f06fcf(j,s,1,a(1,j),1)
          End Do

!         Print the scaled matrix

          ifail = 0
          Call x04caf('Upper','Non-unit',n,n,a,lda,'Scaled matrix',ifail)

        End If
      End If
```

```
99999 Format (1X,A,I4,A)
99998 Format (1X,2(A,1P,E8.1))
99997 Format ((1X,1P,7E11.1))
    End Program f07fffe
```

9.2 Program Data

```
F07FFF Example Program Data
  4                                           :Value of N
  4.16D+00  -3.12D+05   0.56D+00  -0.10D+00
             5.03D+10  -0.83D+05   1.18D+05
                        0.76D+00   0.34D+00
                                   1.18D+00 :End of matrix A
```

9.3 Program Results

```
 F07FFF Example Program Results

 Matrix A
             1            2            3            4
 1    4.1600E+00  -3.1200E+05   5.6000E-01  -1.0000E-01
 2                 5.0300E+10  -8.3000E+04   1.1800E+05
 3                              7.6000E-01   3.4000E-01
 4                                           1.1800E+00

 SCOND = 3.9E-06, AMAX = 5.0E+10

 Diagonal scaling factors
     4.9E-01    4.5E-06    1.1E+00    9.2E-01

 Scaled matrix
           1          2          3          4
 1      1.0000    -0.6821     0.3149    -0.0451
 2                 1.0000    -0.4245     0.4843
 3                            1.0000     0.3590
 4                                       1.0000
```