NAG Library Routine Document

F07ARF (ZGETRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07ARF (ZGETRF) computes the LU factorization of a complex m by n matrix.

2 Specification

```
SUBROUTINE F07ARF (M, N, A, LDA, IPIV, INFO)

INTEGER                 M, N, LDA, IPIV(min(M,N)), INFO
COMPLEX (KIND=nag_wp) A(LDA,*)
```

The routine may be called by its LAPACK name ***zgetrf***.

3 Description

F07ARF (ZGETRF) forms the LU factorization of a complex m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square $(m = n)$, and both L and U are triangular. The routine uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: M – INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: M ≥ 0.

2: N – INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the m by n matrix A.

On exit: the factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07ARF (ZGETRF) is called.

Constraint: LDA $\geq \max(1, \mathrm{M})$.

5: IPIV(min(M, N)) – INTEGER array *Output*

On exit: the pivot indices that define the permutation matrix. At the ith step, if IPIV$(i) > i$ then row i of the matrix A was interchanged with row IPIV(i), for $i = 1, 2, \ldots, \min(m, n)$. IPIV$(i) \leq i$ indicates that, at the ith step, a row interchange was not required.

6: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating point operations is approximately $\frac{8}{3}n^3$ if $m = n$ (the usual case), $\frac{4}{3}n^2(3m - n)$ if $m > n$ and $\frac{4}{3}m^2(3n - m)$ if $m < n$.

A call to this routine with $m = n$ may be followed by calls to the routines:

F07ASF (ZGETRS) to solve $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$;

F07AUF (ZGECON) to estimate the condition number of A;

F07AWF (ZGETRI) to compute the inverse of A.

The real analogue of this routine is F07ADF (DGETRF).

9 Example

This example computes the LU factorization of the matrix A, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}.$$

9.1 Program Text

```
    Program f07arfe

!     F07ARF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zgetrf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, m, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: a(:,:)
      Integer, Allocatable             :: ipiv(:)
      Character (1)                    :: clabs(1), rlabs(1)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: min
!     .. Executable Statements ..
      Write (nout,*) 'F07ARF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) m, n
      lda = m
      Allocate (a(lda,n),ipiv(n))

!     Read A from data file

      Read (nin,*)(a(i,1:n),i=1,m)

!     Factorize A

!     The NAG name equivalent of zgetrf is f07arf
      Call zgetrf(m,n,a,lda,ipiv,info)

!     Print details of factorization

      Write (nout,*)
      Flush (nout)

!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04dbf('General',' ',m,n,a,lda,'Bracketed','F7.4', &
        'Details of factorization','Integer',rlabs,'Integer',clabs,80,0,ifail)

!     Print pivot indices

      Write (nout,*)
      Write (nout,*) 'IPIV'
      Write (nout,99999) ipiv(1:min(m,n))

      If (info/=0) Write (nout,*) 'The factor U is singular'

99999 Format ((1X,I12,3I18))
    End Program f07arfe
```

9.2 Program Data

```
F07ARF Example Program Data
  4  4                                                    :Values of M and N
 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67)  :End of matrix A
```

9.3 Program Results

```
 F07ARF Example Program Results

 Details of factorization
                     1                  2                  3                  4
 1  (-3.2900,-2.3900) (-1.9100, 4.4200) (-0.1400,-1.3500) ( 1.7200, 1.3500)
 2  ( 0.2376, 0.2560) ( 4.8952,-0.7114) (-0.4623, 1.6966) ( 1.2269, 0.6190)
 3  (-0.1020,-0.7010) (-0.6691, 0.3689) (-5.1414,-1.1300) ( 0.9983, 0.3850)
 4  (-0.5359, 0.2707) (-0.2040, 0.8601) ( 0.0082, 0.1211) ( 0.1482,-0.1252)

 IPIV
           3                  2                  3                  4
```