NAG Library Routine Document

F07AAF (DGESV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07AAF (DGESV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07AAF (N, NRHS, A, LDA, IPIV, B, LDB, INFO)

INTEGER            N, NRHS, LDA, IPIV(N), LDB, INFO
REAL (KIND=nag_wp) A(LDA,*), B(LDB,*)
```

The routine may be called by its LAPACK name ***dgesv***.

3 Description

F07AAF (DGESV) uses the LU decomposition with partial pivoting and row interchanges to factor A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: N ≥ 0.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: NRHS ≥ 0.

3: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least max(1, N).

On entry: the n by n coefficient matrix A.

On exit: the factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07AAF (DGESV) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: IPIV(N) – INTEGER array *Output*

On exit: if no constraints are violated, the pivot indices that define the permutation matrix P; at the ith step row i of the matrix was interchanged with row IPIV(i). IPIV(i) $= i$ indicates a row interchange was not required.

6: B(LDB,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07AAF (DGESV) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies the equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1}$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of F07AAF (DGESV), F07AGF (DGECON) can be used to estimate the condition number of A and F07AHF (DGERFS) can be used to obtain approximate error bounds. Alternatives to F07AAF (DGESV), which return condition and error estimates directly are F04BAF and F07ABF (DGESVX).

8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogue of this routine is F07ANF (ZGESV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the general matrix

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 9.52 \\ 24.35 \\ 0.77 \\ -6.22 \end{pmatrix}.$$

Details of the LU factorization of A are also output.

9.1 Program Text

```
    Program f07aafe

!     F07AAF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dgesv, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, ldb, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:), b(:)
      Integer, Allocatable             :: ipiv(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07AAF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      ldb = n
      Allocate (a(lda,n),b(ldb),ipiv(n))

!     Read A and B from data file

      Read (nin,*)(a(i,1:n),i=1,n)
      Read (nin,*) b(1:n)

!     Solve the equations Ax = b for x

!     The NAG name equivalent of dgesv is f07aaf
      Call dgesv(n,1,a,lda,ipiv,b,ldb,info)
```

```
      If (info==0) Then

!       Print solution

        Write (nout,*) 'Solution'
        Write (nout,99999) b(1:n)

!       Print details of factorization

        Write (nout,*)
        Flush (nout)

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf('General',' ',n,n,a,lda,'Details of factorization',ifail)

!       Print pivot indices

        Write (nout,*)
        Write (nout,*) 'Pivot indices'
        Write (nout,99998) ipiv(1:n)

      Else
        Write (nout,99997) 'The (', info, ',', info, ')', &
          ' element of the factor U is zero'
      End If

99999 Format ((3X,7F11.4))
99998 Format ((3X,7I11))
99997 Format (1X,A,I3,A,I3,A,A)
    End Program f07aafe
```

9.2 Program Data

```
F07AAF Example Program Data

  4                          :Value of N

  1.80   2.88   2.05  -0.89
  5.25  -2.95  -0.95  -3.80
  1.58  -2.69  -2.90  -1.04
 -1.11  -0.66  -0.59   0.80   :End of matrix A

  9.52  24.35   0.77  -6.22   :End of vector b
```

9.3 Program Results

```
 F07AAF Example Program Results

 Solution
        1.0000    -1.0000     3.0000    -5.0000

 Details of factorization
             1          2          3          4
 1      5.2500    -2.9500    -0.9500    -3.8000
 2      0.3429     3.8914     2.3757     0.4129
 3      0.3010    -0.4631    -1.5139     0.2948
 4     -0.2114    -0.3299     0.0047     0.1314

 Pivot indices
             2          2          3          4
```