

NAG Library Routine Document

D03NEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

D03NEF computes average values of a continuous function of time over the remaining life of an option. It is used together with D03NDF to value options with time-dependent parameters.

2 Specification

```
SUBROUTINE D03NEF (T0, TMAT, NTD, TD, PHID, PHIIV, WORK, LWORK, IFAIL)
INTEGER          NTD, LWORK, IFAIL
REAL (KIND=nag_wp) T0, TMAT, TD(NTD), PHID(NTD), PHIIV(3), WORK(LWORK)
```

3 Description

D03NEF computes the quantities

$$\phi(t_0), \quad \hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left(\frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}$$

from a given set of values PHID of a continuous time-dependent function $\phi(t)$ at a set of discrete points TD, where t_0 is the current time and T is the maturity time. Thus $\hat{\phi}$ and $\bar{\phi}$ are first and second order averages of ϕ over the remaining life of an option.

The routine may be used in conjunction with D03NDF in order to value an option in the case where the risk-free interest rate r , the continuous dividend q , or the stock volatility σ is time-dependent and is described by values at a set of discrete times (see Section 8.2). This is illustrated in Section 9.

4 References

None.

5 Parameters

- | | | |
|----|---|--------------|
| 1: | T0 – REAL (KIND=nag_wp) <i>On entry:</i> the current time t_0 . <i>Constraint:</i> $TD(1) \leq T0 \leq TD(NTD)$. | <i>Input</i> |
| 2: | TMAT – REAL (KIND=nag_wp) <i>On entry:</i> the maturity time T . <i>Constraint:</i> $TD(1) \leq TMAT \leq TD(NTD)$. | <i>Input</i> |
| 3: | NTD – INTEGER <i>On entry:</i> the number of discrete times at which ϕ is given. <i>Constraint:</i> $NTD \geq 2$. | <i>Input</i> |

- 4: TD(NTD) – REAL (KIND=nag_wp) array Input
On entry: the discrete times at which ϕ is specified.
Constraint: TD(1) < TD(2) < ... < TD(NTD).
- 5: PHID(NTD) – REAL (KIND=nag_wp) array Input
On entry: PHID(i) must contain the value of ϕ at time TD(i), for $i = 1, 2, \dots, \text{NTD}$.
- 6: PHI(3) – REAL (KIND=nag_wp) array Output
On exit: PHI(1) contains the value of ϕ interpolated to t_0 , PHI(2) contains the first-order average $\hat{\phi}$ and PHI(3) contains the second-order average $\bar{\phi}$, where:
- $$\hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left(\frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}.$$
- 7: WORK(LWORK) – REAL (KIND=nag_wp) array Workspace
8: LWORK – INTEGER Input
On entry: the dimension of the array WORK as declared in the (sub)program from which D03NEF is called.
Constraint: LWORK $\geq 9 \times \text{NTD} + 24$.
- 9: IFAIL – INTEGER Input/Output
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

- On entry, T0 lies outside the range [TD(1), TD(NTD)],
- or T0 lies outside the range [TD(1), TD(NTD)],
- or NTD < 2,
- or TD badly ordered,
- or LWORK < 9 × NTD + 24.

IFAIL = 2

Unexpected failure in internal call to E01BAF or E02BBF.

7 Accuracy

If $\phi \in C^4[t_0, T]$ then the error in the approximation of $\phi(t_0)$ and $\hat{\phi}$ is $O(H^4)$, where $H = \max_i (T(i+1) - T(i))$, for $i = 1, 2, \dots, \text{NTD} - 1$. The approximation is exact for polynomials of degree up to 3.

The third quantity $\bar{\phi}$ is $O(H^2)$, and exact for linear functions.

8 Further Comments

8.1 Timing

The time taken is proportional to NTD.

8.2 Use with D03NDF

Suppose you wish to evaluate the analytic solution of the Black–Scholes equation in the case when the risk-free interest rate r is a known function of time, and is represented as a set of values at discrete times. A call to D03NEF providing these values in PHID produces an output array PHIIV suitable for use as the argument R in a subsequent call to D03NDF.

Time-dependent values of the continuous dividend Q and the volatility σ may be handled in the same way.

8.3 Algorithmic Details

The NTD data points are fitted with a cubic B-spline using the routine E01BAF. Evaluation is then performed using E02BBF, and the definite integrals are computed using direct integration of the cubic splines in each interval. The special case of $T = t_0$ is handled by interpolating ϕ at that point.

9 Example

This example demonstrates the use of the routine in conjunction with D03NDF to solve the Black–Scholes equation for valuation of a 5-month American call option on a non-dividend-paying stock with an exercise price of \$50. The risk-free interest rate varies linearly with time and the stock volatility has a quadratic variation. Since these functions are integrated exactly by D03NEF the solution of the Black–Scholes equation by D03NDF is also exact.

The option is valued at a range of times and stock prices.

9.1 Program Text

```
! D03NEF Example Program Text
! Mark 24 Release. NAG Copyright 2012.

Module d03nefe_mod

! D03NEF Example Program Module:
! Parameters and User-defined Routines

! .. Use Statements ..
Use nag_library, Only: nag_wp
! .. Implicit None Statement ..
Implicit None
! .. Parameters ..
Integer, Parameter :: nin = 5, nout = 6
Contains
Subroutine print_greek(ns,nt,tmat,s,t,grname,greek)

! .. Scalar Arguments ..
Real (Kind=nag_wp), Intent (In) :: tmat
Integer, Intent (In) :: ns, nt
Character (*), Intent (In) :: grname
! .. Array Arguments ..
Real (Kind=nag_wp), Intent (In) :: greek(ns,nt), s(ns), t(nt)
```

```

!      .. Local Scalars ..
      Integer                                :: i, j
!      .. Intrinsic Procedures ..
      Intrinsic                              :: len
!      .. Executable Statements ..
      Write (nout,*)
      Write (nout,*) grname
      Write (nout,*) ('-',i=1,len(grname))
      Write (nout,*) ' Stock Price | Time to Maturity (months)'
      Write (nout,99999) '|', (12.0_nag_wp*(tmat-t(i)),i=1,nt)
      Write (nout,*) ' -----', ('-----',i=1,nt)
      Do i = 1, ns
         Write (nout,99998) s(i), '|', (greek(i,j),j=1,nt)
      End Do

      Return

99999  Format (16X,A,1X,12(1P,E12.4))
99998  Format (1X,1P,E12.4,3X,A,1X,12(1P,E12.4))
      End Subroutine print_greek
      End Module d03nefe_mod

Program d03nefe

!      D03NEF Example Main Program

!      .. Use Statements ..
      Use nag_library, Only: d03ndf, d03nef, nag_wp
      Use d03nefe_mod, Only: nin, nout, print_greek
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Logical, Parameter                      :: gprnt(5) = .True.
!      .. Local Scalars ..
      Real (Kind=nag_wp)                     :: ds, dt, tmat, x
      Integer                                 :: i, ifail, j, kopt, lwork, ns,      &
      nt, ntd
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable        :: delta(:,,:), f(:,,:), gamma(:,,:), &
      lambda(:,,:), rd(:,,:), rho(:,,:),    &
      s(:,), sigd(:,), t(:,), td(:,),      &
      theta(:,), work(:)
      Real (Kind=nag_wp)                     :: q(3), ra(3), siga(3)
      Logical                                 :: tdpar(3)
!      .. Intrinsic Procedures ..
      Intrinsic                              :: real
!      .. Executable Statements ..
      Write (nout,*) 'D03NEF Example Program Results'
      Write (nout,*)

!      Skip heading in data file
      Read (nin,*)
      Read (nin,*) ns, nt, ntd
      lwork = 9*ntd + 24

      Allocate (delta(ns,nt),f(ns,nt),gamma(ns,nt),lambda(ns,nt),rd(ntd), &
      rho(ns,nt),s(ns),sigd(ntd),t(nt),td(ntd),theta(ns,nt),work(lwork))

!      Read problem parameters

      Read (nin,*) kopt
      Read (nin,*) x
      Read (nin,*) tmat
      Read (nin,*) s(1), s(ns)
      Read (nin,*) t(1), t(nt)
      Read (nin,*) td(1:ntd)
      Read (nin,*) rd(1:ntd)
      Read (nin,*) sigd(1:ntd)
      Read (nin,*) tdpar(1:3)
      Read (nin,*) q(1)

```

```

If (ns<2) Then
  Write (nout,*) 'NS invalid.'
Else If (nt<2) Then
  Write (nout,*) 'NT invalid.'
Else

  ds = (s(ns)-s(1))/real(ns-1,kind=nag_wp)
  dt = (t(nt)-t(1))/real(nt-1,kind=nag_wp)

!   Loop over times

  Do j = 1, nt

    t(j) = t(1) + real(j-1,kind=nag_wp)*dt

!   Find average values of r and sigma
    ifail = 0
    Call d03nef(t(j),tmat,ntd,td,rd,ra,work,lwork,ifail)

    ifail = 0
    Call d03nef(t(j),tmat,ntd,td,sigd,siga,work,lwork,ifail)

!   Loop over stock prices

    Do i = 1, ns

      s(i) = s(1) + real(i-1,kind=nag_wp)*ds

!   Evaluate analytic solution of Black-Scholes equation
      ifail = 0
      Call d03ndf(kopt,x,s(i),t(j),tmat,tdpar,ra,q,siga,f(i,j), &
        theta(i,j),delta(i,j),gamma(i,j),lambda(i,j),rho(i,j),ifail)

      End Do
    End Do

!   Output option values and possibly Greeks.

    Call print_greek(ns,nt,tmat,s,t,'Option Values',f)

    If (gprnt(1)) Call print_greek(ns,nt,tmat,s,t,'Theta',theta)
    If (gprnt(2)) Call print_greek(ns,nt,tmat,s,t,'Delta',delta)
    If (gprnt(3)) Call print_greek(ns,nt,tmat,s,t,'Gamma',gamma)
    If (gprnt(4)) Call print_greek(ns,nt,tmat,s,t,'Lambda',lambda)
    If (gprnt(5)) Call print_greek(ns,nt,tmat,s,t,'Rho',rho)

  End If

End Program d03nefe

```

9.2 Program Data

```

D03NEF Example Program Data
21 4 6           : ns, nt, ntd
2               : kopt
50.             : x
0.4166667      : tmat
0.0 100.        : s(1), s(ns)
0.0 0.125       : t(1), t(nt)
0.00 0.10 0.20
0.30 0.40 0.50   : td
0.10 0.11 0.12
0.13 0.14 0.15   : rd
0.30 0.46 0.54
0.54 0.46 0.30   : sigd
.TRUE. .FALSE. .TRUE. : tdpar
0.0             : q(1)

```

9.3 Program Results

D03NEF Example Program Results

Option Values

```

-----
Stock Price | Time to Maturity (months)
            | 5.0000E+00 4.5000E+00 4.0000E+00 3.5000E+00
-----
0.0000E+00 | 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
5.0000E+00 | 3.3671E-13 7.7404E-14 7.3210E-15 2.0179E-16
1.0000E+01 | 5.2088E-07 2.4281E-07 7.2216E-08 1.1540E-08
1.5000E+01 | 2.6607E-04 1.6753E-04 8.0943E-05 2.7179E-05
2.0000E+01 | 8.9697E-03 6.6505E-03 4.1780E-03 2.0942E-03
2.5000E+01 | 8.3647E-02 6.8467E-02 5.0375E-02 3.2105E-02
3.0000E+01 | 3.8221E-01 3.3331E-01 2.7117E-01 2.0119E-01
3.5000E+01 | 1.1298E+00 1.0275E+00 8.9292E-01 7.3146E-01
4.0000E+01 | 2.5164E+00 2.3541E+00 2.1380E+00 1.8699E+00
4.5000E+01 | 4.6249E+00 4.4110E+00 4.1267E+00 3.7700E+00
5.0000E+01 | 7.4287E+00 7.1797E+00 6.8531E+00 6.4449E+00
5.5000E+01 | 1.0830E+01 1.0564E+01 1.0221E+01 9.7996E+00
6.0000E+01 | 1.4707E+01 1.4436E+01 1.4097E+01 1.3689E+01
6.5000E+01 | 1.8937E+01 1.8671E+01 1.8348E+01 1.7968E+01
7.0000E+01 | 2.3421E+01 2.3164E+01 2.2860E+01 2.2514E+01
7.5000E+01 | 2.8080E+01 2.7833E+01 2.7550E+01 2.7234E+01
8.0000E+01 | 3.2857E+01 3.2620E+01 3.2354E+01 3.2064E+01
8.5000E+01 | 3.7713E+01 3.7484E+01 3.7233E+01 3.6963E+01
9.0000E+01 | 4.2620E+01 4.2398E+01 4.2158E+01 4.1904E+01
9.5000E+01 | 4.7561E+01 4.7344E+01 4.7112E+01 4.6868E+01
1.0000E+02 | 5.2523E+01 5.2310E+01 5.2084E+01 5.1848E+01
    
```

Theta

```

-----
Stock Price | Time to Maturity (months)
            | 5.0000E+00 4.5000E+00 4.0000E+00 3.5000E+00
-----
0.0000E+00 | 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
5.0000E+00 | -8.9082E-12 -3.4507E-12 -5.0884E-13 -2.1236E-14
1.0000E+01 | -7.2097E-06 -5.5915E-06 -2.5721E-06 -6.1830E-07
1.5000E+01 | -2.2499E-03 -2.3259E-03 -1.7227E-03 -8.6349E-04
2.0000E+01 | -4.9483E-02 -5.9355E-02 -5.6562E-02 -4.1921E-02
2.5000E+01 | -3.1200E-01 -4.0620E-01 -4.4765E-01 -4.1683E-01
3.0000E+01 | -9.8578E-01 -1.3408E+00 -1.6092E+00 -1.7186E+00
3.5000E+01 | -2.0479E+00 -2.8395E+00 -3.5745E+00 -4.1390E+00
4.0000E+01 | -3.2501E+00 -4.5165E+00 -5.8147E+00 -7.0323E+00
4.5000E+01 | -4.3144E+00 -5.9349E+00 -7.6762E+00 -9.4488E+00
5.0000E+01 | -5.0802E+00 -6.8543E+00 -8.7919E+00 -1.0815E+01
5.5000E+01 | -5.5225E+00 -7.2603E+00 -9.1500E+00 -1.1104E+01
6.0000E+01 | -5.7006E+00 -7.2722E+00 -8.9491E+00 -1.0625E+01
6.5000E+01 | -5.7014E+00 -7.0446E+00 -8.4366E+00 -9.7565E+00
7.0000E+01 | -5.6037E+00 -6.7093E+00 -7.8142E+00 -8.7951E+00
7.5000E+01 | -5.4653E+00 -6.3555E+00 -7.2107E+00 -7.9170E+00
8.0000E+01 | -5.3218E+00 -6.0329E+00 -6.6903E+00 -7.1974E+00
8.5000E+01 | -5.1920E+00 -5.7627E+00 -6.2736E+00 -6.6481E+00
9.0000E+01 | -5.0833E+00 -5.5487E+00 -5.9563E+00 -6.2492E+00
9.5000E+01 | -4.9969E+00 -5.3857E+00 -5.7234E+00 -5.9700E+00
1.0000E+02 | -4.9306E+00 -5.2651E+00 -5.5570E+00 -5.7797E+00
    
```

Delta

```

-----
Stock Price | Time to Maturity (months)
            | 5.0000E+00 4.5000E+00 4.0000E+00 3.5000E+00
-----
0.0000E+00 | 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
5.0000E+00 | 1.6086E-12 3.8832E-13 3.9572E-14 1.2111E-15
1.0000E+01 | 8.9933E-07 4.3972E-07 1.4063E-07 2.4884E-08
1.5000E+01 | 2.3975E-04 1.5810E-04 8.1943E-05 3.0366E-05
2.0000E+01 | 4.9150E-03 3.8095E-03 2.5596E-03 1.4100E-03
2.5000E+01 | 3.0345E-02 2.5906E-02 2.0311E-02 1.4153E-02
3.0000E+01 | 9.6991E-02 8.7980E-02 7.5946E-02 6.1231E-02
    
```

| | | | | | |
|------------|--|------------|------------|------------|------------|
| 3.5000E+01 | | 2.0863E-01 | 1.9675E-01 | 1.8053E-01 | 1.5957E-01 |
| 4.0000E+01 | | 3.4875E-01 | 3.3719E-01 | 3.2158E-01 | 3.0109E-01 |
| 4.5000E+01 | | 4.9361E-01 | 4.8480E-01 | 4.7356E-01 | 4.5924E-01 |
| 5.0000E+01 | | 6.2450E-01 | 6.1931E-01 | 6.1363E-01 | 6.0735E-01 |
| 5.5000E+01 | | 7.3200E-01 | 7.3000E-01 | 7.2907E-01 | 7.2954E-01 |
| 6.0000E+01 | | 8.1439E-01 | 8.1462E-01 | 8.1681E-01 | 8.2145E-01 |
| 6.5000E+01 | | 8.7440E-01 | 8.7589E-01 | 8.7961E-01 | 8.8602E-01 |
| 7.0000E+01 | | 9.1650E-01 | 9.1850E-01 | 9.2260E-01 | 9.2911E-01 |
| 7.5000E+01 | | 9.4522E-01 | 9.4726E-01 | 9.5107E-01 | 9.5679E-01 |
| 8.0000E+01 | | 9.6441E-01 | 9.6624E-01 | 9.6946E-01 | 9.7406E-01 |
| 8.5000E+01 | | 9.7704E-01 | 9.7856E-01 | 9.8111E-01 | 9.8461E-01 |
| 9.0000E+01 | | 9.8526E-01 | 9.8646E-01 | 9.8839E-01 | 9.9094E-01 |
| 9.5000E+01 | | 9.9057E-01 | 9.9148E-01 | 9.9290E-01 | 9.9470E-01 |
| 1.0000E+02 | | 9.9397E-01 | 9.9464E-01 | 9.9567E-01 | 9.9691E-01 |

Gamma

| Stock Price | | Time to Maturity (months) | | | |
|-------------|--|---------------------------|------------|------------|------------|
| | | 5.0000E+00 | 4.5000E+00 | 4.0000E+00 | 3.5000E+00 |
| 0.0000E+00 | | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 5.0000E+00 | | 7.2334E-12 | 1.8390E-12 | 2.0276E-13 | 6.9267E-15 |
| 1.0000E+01 | | 1.4139E-06 | 7.2829E-07 | 2.5205E-07 | 4.9786E-08 |
| 1.5000E+01 | | 1.8932E-04 | 1.3153E-04 | 7.3756E-05 | 3.0494E-05 |
| 2.0000E+01 | | 2.2528E-03 | 1.8392E-03 | 1.3360E-03 | 8.2017E-04 |
| 2.5000E+01 | | 8.6933E-03 | 7.8126E-03 | 6.6135E-03 | 5.1251E-03 |
| 3.0000E+01 | | 1.8099E-02 | 1.7264E-02 | 1.6056E-02 | 1.4350E-02 |
| 3.5000E+01 | | 2.5953E-02 | 2.5691E-02 | 2.5315E-02 | 2.4683E-02 |
| 4.0000E+01 | | 2.9260E-02 | 2.9618E-02 | 3.0194E-02 | 3.0968E-02 |
| 4.5000E+01 | | 2.8046E-02 | 2.8736E-02 | 2.9814E-02 | 3.1368E-02 |
| 5.0000E+01 | | 2.4005E-02 | 2.4715E-02 | 2.5793E-02 | 2.7346E-02 |
| 5.5000E+01 | | 1.8950E-02 | 1.9500E-02 | 2.0296E-02 | 2.1401E-02 |
| 6.0000E+01 | | 1.4105E-02 | 1.4449E-02 | 1.4903E-02 | 1.5476E-02 |
| 6.5000E+01 | | 1.0054E-02 | 1.0221E-02 | 1.0396E-02 | 1.0555E-02 |
| 7.0000E+01 | | 6.9401E-03 | 6.9861E-03 | 6.9806E-03 | 6.8890E-03 |
| 7.5000E+01 | | 4.6779E-03 | 4.6538E-03 | 4.5552E-03 | 4.3505E-03 |
| 8.0000E+01 | | 3.0978E-03 | 3.0414E-03 | 2.9096E-03 | 2.6800E-03 |
| 8.5000E+01 | | 2.0250E-03 | 1.9598E-03 | 1.8291E-03 | 1.6205E-03 |
| 9.0000E+01 | | 1.3114E-03 | 1.2499E-03 | 1.1365E-03 | 9.6637E-04 |
| 9.5000E+01 | | 8.4362E-04 | 7.9138E-04 | 7.0024E-04 | 5.7052E-04 |
| 1.0000E+02 | | 5.4033E-04 | 4.9856E-04 | 4.2893E-04 | 3.3442E-04 |

Lambda

| Stock Price | | Time to Maturity (months) | | | |
|-------------|--|---------------------------|------------|------------|------------|
| | | 5.0000E+00 | 4.5000E+00 | 4.0000E+00 | 3.5000E+00 |
| 0.0000E+00 | | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 5.0000E+00 | | 3.6558E-11 | 8.6441E-12 | 8.6672E-13 | 2.6259E-14 |
| 1.0000E+01 | | 2.8583E-05 | 1.3693E-05 | 4.3098E-06 | 7.5495E-07 |
| 1.5000E+01 | | 8.6115E-03 | 5.5645E-03 | 2.8375E-03 | 1.0404E-03 |
| 2.0000E+01 | | 1.8217E-01 | 1.3832E-01 | 9.1376E-02 | 4.9748E-02 |
| 2.5000E+01 | | 1.0984E+00 | 9.1808E-01 | 7.0676E-01 | 4.8574E-01 |
| 3.0000E+01 | | 3.2931E+00 | 2.9214E+00 | 2.4708E+00 | 1.9584E+00 |
| 3.5000E+01 | | 6.4272E+00 | 5.9173E+00 | 5.3025E+00 | 4.5851E+00 |
| 4.0000E+01 | | 9.4643E+00 | 8.9101E+00 | 8.2604E+00 | 7.5135E+00 |
| 4.5000E+01 | | 1.1481E+01 | 1.0941E+01 | 1.0323E+01 | 9.6323E+00 |
| 5.0000E+01 | | 1.2132E+01 | 1.1617E+01 | 1.1026E+01 | 1.0367E+01 |
| 5.5000E+01 | | 1.1588E+01 | 1.1091E+01 | 1.0498E+01 | 9.8169E+00 |
| 6.0000E+01 | | 1.0265E+01 | 9.7801E+00 | 9.1734E+00 | 8.4486E+00 |
| 6.5000E+01 | | 8.5872E+00 | 8.1198E+00 | 7.5104E+00 | 6.7621E+00 |
| 7.0000E+01 | | 6.8747E+00 | 6.4363E+00 | 5.8487E+00 | 5.1188E+00 |
| 7.5000E+01 | | 5.3194E+00 | 4.9219E+00 | 4.3812E+00 | 3.7109E+00 |
| 8.0000E+01 | | 4.0081E+00 | 3.6599E+00 | 3.1840E+00 | 2.6009E+00 |
| 8.5000E+01 | | 2.9578E+00 | 2.6623E+00 | 2.2597E+00 | 1.7754E+00 |
| 9.0000E+01 | | 2.1474E+00 | 1.9036E+00 | 1.5741E+00 | 1.1870E+00 |
| 9.5000E+01 | | 1.5392E+00 | 1.3429E+00 | 1.0806E+00 | 7.8078E-01 |
| 1.0000E+02 | | 1.0923E+00 | 9.3740E-01 | 7.3341E-01 | 5.0711E-01 |

Rho

| Stock Price | Time to Maturity (months) | | | |
|-------------|---------------------------|------------|------------|------------|
| | 5.0000E+00 | 4.5000E+00 | 4.0000E+00 | 3.5000E+00 |
| 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| 5.0000E+00 | 3.2110E-12 | 6.9908E-13 | 6.3513E-14 | 1.7073E-15 |
| 1.0000E+01 | 3.5302E-06 | 1.5579E-06 | 4.4470E-07 | 6.9214E-08 |
| 1.5000E+01 | 1.3876E-03 | 8.2648E-04 | 3.8273E-04 | 1.2492E-04 |
| 2.0000E+01 | 3.7221E-02 | 2.6077E-02 | 1.5671E-02 | 7.6142E-03 |
| 2.5000E+01 | 2.8124E-01 | 2.1719E-01 | 1.5247E-01 | 9.3836E-02 |
| 3.0000E+01 | 1.0531E+00 | 8.6478E-01 | 6.6907E-01 | 4.7709E-01 |
| 3.5000E+01 | 2.5718E+00 | 2.1971E+00 | 1.8086E+00 | 1.4156E+00 |
| 4.0000E+01 | 4.7641E+00 | 4.1750E+00 | 3.5750E+00 | 2.9673E+00 |
| 4.5000E+01 | 7.3281E+00 | 6.5270E+00 | 5.7279E+00 | 4.9280E+00 |
| 5.0000E+01 | 9.9152E+00 | 8.9196E+00 | 7.9427E+00 | 6.9774E+00 |
| 5.5000E+01 | 1.2262E+01 | 1.1095E+01 | 9.9592E+00 | 8.8448E+00 |
| 6.0000E+01 | 1.4232E+01 | 1.2915E+01 | 1.1637E+01 | 1.0383E+01 |
| 6.5000E+01 | 1.5791E+01 | 1.4348E+01 | 1.2942E+01 | 1.1557E+01 |
| 7.0000E+01 | 1.6973E+01 | 1.5424E+01 | 1.3907E+01 | 1.2403E+01 |
| 7.5000E+01 | 1.7838E+01 | 1.6204E+01 | 1.4594E+01 | 1.2987E+01 |
| 8.0000E+01 | 1.8457E+01 | 1.6755E+01 | 1.5067E+01 | 1.3376E+01 |
| 8.5000E+01 | 1.8890E+01 | 1.7135E+01 | 1.5387E+01 | 1.3629E+01 |
| 9.0000E+01 | 1.9189E+01 | 1.7393E+01 | 1.5599E+01 | 1.3790E+01 |
| 9.5000E+01 | 1.9393E+01 | 1.7567E+01 | 1.5738E+01 | 1.3891E+01 |
| 1.0000E+02 | 1.9531E+01 | 1.7683E+01 | 1.5827E+01 | 1.3954E+01 |

Example Program 1
Option Values and Derivatives at 5 Months to Maturity



