NAG Library Routine Document

D02UZF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

D02UZF returns the value of the kth Chebyshev polynomial evaluated at a point $x \in [-1, 1]$. D02UZF is primarily a utility routine for use by the Chebyshev boundary value problem solvers.

2 Specification

```
SUBROUTINE D02UZF (K, X, T, IFAIL)

INTEGER          K, IFAIL
REAL (KIND=nag_wp) X, T
```

3 Description

D02UZF returns the value, T, of the kth Chebyshev polynomial evaluated at a point $x \in [-1, 1]$; that is, $T = \cos(k \times \arccos(x))$.

4 References

Trefethen L N (2000) *Spectral Methods in MATLAB* SIAM

5 Parameters

1: K – INTEGER   *Input*

    On entry: the order of the Chebyshev polynomial.

    Constraint: $K \geq 0$.

2: X – REAL (KIND=nag_wp)   *Input*

    On entry: the point at which to evaluate the polynomial.

    Constraint: $-1.0 \leq X \leq 1.0$.

3: T – REAL (KIND=nag_wp)   *Output*

    On exit: the value, T, of the Chebyshev polynomial order k evaluated at x.

4: IFAIL – INTEGER   *Input/Output*

    On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

    For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

    On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

IFAIL = 1

On entry, $K < 0$.

IFAIL = 2

On entry, $X < -1.0$ or $X > 1.0$.

7 Accuracy

The accuracy should be close to *machine precision*.

8 Further Comments

None.

9 Example

A set of Chebyshev coefficients is obtained for the function $x + \exp(-x)$ defined on $[-0.24 \times \pi, 0.5 \times \pi]$ using D02UCF. At each of a set of new grid points in the domain of the function D02UZF is used to evaluate each Chebshev polynomial in the series representation. The values obtained are multiplied to the Chebyshev coefficients and summed to obtain approximations to the given function at the new grid points.

9.1 Program Text

```
!   D02UZF Example Program Text
!   Mark 24 Release. NAG Copyright 2012.

    Module d02uzfe_mod

!     D02UZF Example Program Module:
!            Parameters and User-defined Routines

!     .. Use Statements ..
      Use nag_library, Only: nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Real (Kind=nag_wp), Parameter     :: one = 1.0_nag_wp
      Real (Kind=nag_wp), Parameter     :: two = 2.0_nag_wp
      Real (Kind=nag_wp), Parameter     :: zero = 0.0_nag_wp
      Integer, Parameter                :: nin = 5, nout = 6
      Logical, Parameter                :: reqerr = .False.
!     .. Local Scalars ..
      Real (Kind=nag_wp)                :: a, b
    Contains
      Function exact(x)

!       .. Function Return Value ..
        Real (Kind=nag_wp)              :: exact
!       .. Scalar Arguments ..
        Real (Kind=nag_wp), Intent (In) :: x
!       .. Intrinsic Procedures ..
        Intrinsic                       :: exp
!       .. Executable Statements ..
        exact = x + exp(-x)
        Return
      End Function exact
    End Module d02uzfe_mod
    Program d02uzfe

!     D02UZF Example Main Program
```

```
!        .. Use Statements ..
        Use nag_library, Only: d02uaf, d02ucf, d02uzf, nag_wp, x01aaf, x02ajf
        Use d02uzfe_mod, Only: a, b, exact, nin, nout, one, reqerr, two, zero
!        .. Implicit None Statement ..
        Implicit None
!        .. Local Scalars ..
        Real (Kind=nag_wp)                     :: deven, dmap, fseries, pi, t,    &
                                                  teneps, uerr, xeven, xmap
        Integer                                :: i, ifail, k, m, n
!        .. Local Arrays ..
        Real (Kind=nag_wp), Allocatable       :: c(:), f(:), x(:)
!        .. Intrinsic Procedures ..
        Intrinsic                              :: abs, int, max, min, real
!        .. Executable Statements ..
        Write (nout,*) ' D02UZF Example Program Results '
        Write (nout,*)

        Read (nin,*)
        Read (nin,*) n, m

        Allocate (f(n+1),c(n+1),x(n+1))

!       Set up problem boundary conditions and definition
        pi = x01aaf(pi)
        a = -0.24_nag_wp*pi
        b = pi/two

!       Set up Chebyshev grid
        ifail = 0
        Call d02ucf(n,a,b,x,ifail)

!       Evaluate function on grid and get interpolating Chebyshev coefficients.
        Do i = 1, n + 1
          f(i) = exact(x(i))
        End Do
        ifail = 0
        Call d02uaf(n,f,c,ifail)

!       Evaluate Chebyshev series manually by evaluating each Chebyshev
!       polynomial in turn at new equispaced (m+1) grid points.
!       Chebyshev series on [-1,1] map of [a,b].
        xmap = -one
        dmap = two/real(m-1,kind=nag_wp)
        xeven = a
        deven = (b-a)/real(m-1,kind=nag_wp)

        Write (nout,99999)
        uerr = zero
        Do i = 1, m
          fseries = zero
          Do k = 0, n
            ifail = 0
            Call d02uzf(k,xmap,t,ifail)
            fseries = fseries + c(k+1)*t
          End Do
          uerr = max(uerr,abs(fseries-exact(xeven)))
          Write (nout,99998) xmap, xeven, fseries
          xmap = min(one,xmap+dmap)
          xeven = xeven + deven
        End Do

        If (reqerr) Then
          teneps = 10.0_nag_wp*x02ajf()
          Write (nout,'(//)')
          Write (nout,99997) 10*(int(uerr/teneps)+1)
        End If
```

```
99999 Format (1X,T6,'x_even',T17,'x_map',T28,'Sum')
99998 Format (1X,3F10.4)
99997 Format (1X,'Error in coefficient sum is < ',I8,' * machine precision.')

   End Program d02uzfe
```

9.2   Program Data

```
D02UZF Example Program Data
  16, 9               : N, M
```

9.3   Program Results

```
  D02UZF Example Program Results

    x_even     x_map       Sum
  -1.0000   -0.7540    1.3715
  -0.7500   -0.4634    1.1261
  -0.5000   -0.1728    1.0158
  -0.2500    0.1178    1.0067
   0.0000    0.4084    1.0731
   0.2500    0.6990    1.1961
   0.5000    0.9896    1.3613
   0.7500    1.2802    1.5582
   1.0000    1.5708    1.7787
```

_____