

NAG Library Function Document

nag_binary_con_greeks (s30cbc)

1 Purpose

nag_binary_con_greeks (s30cbc) computes the price of a binary or digital cash-or-nothing option together with its sensitivities (Greeks).

2 Specification

```
#include <nag.h>
#include <nags.h>

void nag_binary_con_greeks (Nag_OrderType order, Nag_CallPut option,
    Integer m, Integer n, const double x[], double s, double k,
    const double t[], double sigma, double r, double q, double p[],
    double delta[], double gamma[], double vega[], double theta[],
    double rho[], double crho[], double vanna[], double charm[],
    double speed[], double colour[], double zomma[], double vomma[],
    NagError *fail)
```

3 Description

nag_binary_con_greeks (s30cbc) computes the price of a binary or digital cash-or-nothing option, together with the Greeks or sensitivities, which are the partial derivatives of the option price with respect to certain of the other input parameters. This option pays a fixed amount, K , at expiration if the option is in-the-money (see Section 2.4 in the s Chapter Introduction). For a strike price, X , underlying asset price, S , and time to expiry, T , the payoff is therefore K , if $S > X$ for a call or $S < X$ for a put. Nothing is paid out when this condition is not met.

The price of a call with volatility, σ , risk-free interest rate, r , and annualised dividend yield, q , is

$$P_{\text{call}} = Ke^{-rT}\Phi(d_2)$$

and for a put,

$$P_{\text{put}} = Ke^{-rT}\Phi(-d_2)$$

where Φ is the cumulative Normal distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp(-y^2/2) dy,$$

and

$$d_2 = \frac{\ln(S/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

The option price $P_{ij} = P(X = X_i, T = T_j)$ is computed for each strike price in a set X_i , $i = 1, 2, \dots, m$, and for each expiry time in a set T_j , $j = 1, 2, \dots, n$.

4 References

Reiner E and Rubinstein M (1991) Unscrambling the binary code *Risk* 4

5 Arguments

- 1: **order** – Nag_OrderType *Input*
On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.
Constraint: **order** = Nag_RowMajor or Nag_ColMajor.
- 2: **option** – Nag_CallPut *Input*
On entry: determines whether the option is a call or a put.
option = Nag_Call
 A call; the holder has a right to buy.
option = Nag_Put
 A put; the holder has a right to sell.
Constraint: **option** = Nag_Call or Nag_Put.
- 3: **m** – Integer *Input*
On entry: the number of strike prices to be used.
Constraint: **m** \geq 1.
- 4: **n** – Integer *Input*
On entry: the number of times to expiry to be used.
Constraint: **n** \geq 1.
- 5: **x[m]** – const double *Input*
On entry: **x**[*i* – 1] must contain X_i , the *i*th strike price, for $i = 1, 2, \dots, \mathbf{m}$.
Constraint: **x**[*i* – 1] $\geq z$ and **x**[*i* – 1] $\leq 1/z$, where $z = \text{nag_real_safe_small_number}$, the safe range parameter, for $i = 1, 2, \dots, \mathbf{m}$.
- 6: **s** – double *Input*
On entry: S , the price of the underlying asset.
Constraint: **s** $\geq z$ and **s** $\leq 1.0/z$, where $z = \text{nag_real_safe_small_number}$, the safe range parameter.
- 7: **k** – double *Input*
On entry: the amount, K , to be paid at expiration if the option is in-the-money, i.e., if **s** $>$ **x**[*i* – 1] when **option** = Nag_Call, or if **s** $<$ **x**[*i* – 1] when **option** = Nag_Put, for $i = 1, 2, \dots, \mathbf{m}$.
Constraint: **k** \geq 0.0.
- 8: **t[n]** – const double *Input*
On entry: **t**[*i* – 1] must contain T_i , the *i*th time, in years, to expiry, for $i = 1, 2, \dots, \mathbf{n}$.
Constraint: **t**[*i* – 1] $\geq z$, where $z = \text{nag_real_safe_small_number}$, the safe range parameter, for $i = 1, 2, \dots, \mathbf{n}$.
- 9: **sigma** – double *Input*
On entry: σ , the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
Constraint: **sigma** $>$ 0.0.

- 10: **r** – double *Input*
On entry: r , the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
Constraint: $r \geq 0.0$.
- 11: **q** – double *Input*
On entry: q , the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
Constraint: $q \geq 0.0$.
- 12: **p**[$m \times n$] – double *Output*
Note: where $\mathbf{P}(i, j)$ appears in this document, it refers to the array element
 $\mathbf{p}[(j - 1) \times m + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{p}[(i - 1) \times n + j - 1]$ when **order** = Nag_RowMajor.
On exit: $\mathbf{P}(i, j)$ contains P_{ij} , the option price evaluated for the strike price x_i at expiry t_j for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.
- 13: **delta**[$m \times n$] – double *Output*
Note: the (i, j) th element of the matrix is stored in
 $\mathbf{delta}[(j - 1) \times m + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{delta}[(i - 1) \times n + j - 1]$ when **order** = Nag_RowMajor.
On exit: the $m \times n$ array **delta** contains the sensitivity, $\frac{\partial P}{\partial S}$, of the option price to change in the price of the underlying asset.
- 14: **gamma**[$m \times n$] – double *Output*
Note: the (i, j) th element of the matrix is stored in
 $\mathbf{gamma}[(j - 1) \times m + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{gamma}[(i - 1) \times n + j - 1]$ when **order** = Nag_RowMajor.
On exit: the $m \times n$ array **gamma** contains the sensitivity, $\frac{\partial^2 P}{\partial S^2}$, of **delta** to change in the price of the underlying asset.
- 15: **vega**[$m \times n$] – double *Output*
Note: where $\mathbf{VEGA}(i, j)$ appears in this document, it refers to the array element
 $\mathbf{vega}[(j - 1) \times m + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{vega}[(i - 1) \times n + j - 1]$ when **order** = Nag_RowMajor.
On exit: $\mathbf{VEGA}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the volatility of the underlying asset, i.e., $\frac{\partial P_{ij}}{\partial \sigma}$, for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.
- 16: **theta**[$m \times n$] – double *Output*
Note: where $\mathbf{THETA}(i, j)$ appears in this document, it refers to the array element
 $\mathbf{theta}[(j - 1) \times m + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{theta}[(i - 1) \times n + j - 1]$ when **order** = Nag_RowMajor.
On exit: $\mathbf{THETA}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in time, i.e., $-\frac{\partial P_{ij}}{\partial T}$, for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$, where $b = r - q$.

17: **rho**[$\mathbf{m} \times \mathbf{n}$] – double Output

Note: where **RHO**(i, j) appears in this document, it refers to the array element

rho[($j - 1$) \times $\mathbf{m} + i - 1$] when **order** = Nag_ColMajor;
rho[($i - 1$) \times $\mathbf{n} + j - 1$] when **order** = Nag_RowMajor.

On exit: **RHO**(i, j), contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the annual risk-free interest rate, i.e., $-\frac{\partial P_{ij}}{\partial r}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

18: **crho**[$\mathbf{m} \times \mathbf{n}$] – double Output

Note: where **CRHO**(i, j) appears in this document, it refers to the array element

crho[($j - 1$) \times $\mathbf{m} + i - 1$] when **order** = Nag_ColMajor;
crho[($i - 1$) \times $\mathbf{n} + j - 1$] when **order** = Nag_RowMajor.

On exit: **CRHO**(i, j), contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the annual cost of carry rate, i.e., $-\frac{\partial P_{ij}}{\partial b}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$, where $b = r - q$.

19: **vanna**[$\mathbf{m} \times \mathbf{n}$] – double Output

Note: where **VANNA**(i, j) appears in this document, it refers to the array element

vanna[($j - 1$) \times $\mathbf{m} + i - 1$] when **order** = Nag_ColMajor;
vanna[($i - 1$) \times $\mathbf{n} + j - 1$] when **order** = Nag_RowMajor.

On exit: **VANNA**(i, j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the volatility of the asset price, i.e., $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial \sigma}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

20: **charm**[$\mathbf{m} \times \mathbf{n}$] – double Output

Note: where **CHARM**(i, j) appears in this document, it refers to the array element

charm[($j - 1$) \times $\mathbf{m} + i - 1$] when **order** = Nag_ColMajor;
charm[($i - 1$) \times $\mathbf{n} + j - 1$] when **order** = Nag_RowMajor.

On exit: **CHARM**(i, j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the time, i.e., $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial T}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

21: **speed**[$\mathbf{m} \times \mathbf{n}$] – double Output

Note: where **SPEED**(i, j) appears in this document, it refers to the array element

speed[($j - 1$) \times $\mathbf{m} + i - 1$] when **order** = Nag_ColMajor;
speed[($i - 1$) \times $\mathbf{n} + j - 1$] when **order** = Nag_RowMajor.

On exit: **SPEED**(i, j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the price of the underlying asset, i.e., $-\frac{\partial \Gamma_{ij}}{\partial S} = -\frac{\partial^3 P_{ij}}{\partial S^3}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

22: **colour**[$\mathbf{m} \times \mathbf{n}$] – double Output

Note: where **COLOUR**(i, j) appears in this document, it refers to the array element

colour[($j - 1$) \times $\mathbf{m} + i - 1$] when **order** = Nag_ColMajor;
colour[($i - 1$) \times $\mathbf{n} + j - 1$] when **order** = Nag_RowMajor.

On exit: **COLOUR**(i, j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the time, i.e., $-\frac{\partial \Gamma_{ij}}{\partial T} = -\frac{\partial^3 P_{ij}}{\partial S \partial T}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

23: **zomma**[**m** × **n**] – double

Output

Note: where **ZOMMA**(*i*, *j*) appears in this document, it refers to the array element

zomma[(*j* – 1) × **m** + *i* – 1] when **order** = Nag_ColMajor;
zomma[(*i* – 1) × **n** + *j* – 1] when **order** = Nag_RowMajor.

On exit: **ZOMMA**(*i*, *j*), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the volatility of the underlying asset, i.e., $-\frac{\partial \Gamma_{ij}}{\partial \sigma} = -\frac{\partial^3 P_{ij}}{\partial \sigma^2}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

24: **vomma**[**m** × **n**] – double

Output

Note: where **VOMMA**(*i*, *j*) appears in this document, it refers to the array element

vomma[(*j* – 1) × **m** + *i* – 1] when **order** = Nag_ColMajor;
vomma[(*i* – 1) × **n** + *j* – 1] when **order** = Nag_RowMajor.

On exit: **VOMMA**(*i*, *j*), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the volatility of the underlying asset, i.e., $-\frac{\partial \Delta_{ij}}{\partial \sigma} = -\frac{\partial^2 P_{ij}}{\partial \sigma^2}$, for $i = 1, 2, \dots, \mathbf{m}$ and $j = 1, 2, \dots, \mathbf{n}$.

25: **fail** – NagError *

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument *<value>* had an illegal value.

NE_INT

On entry, **m** = *<value>*.

Constraint: **m** ≥ 1.

On entry, **n** = *<value>*.

Constraint: **n** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **k** = *<value>*.

Constraint: **k** ≥ 0.0.

On entry, **q** = *<value>*.

Constraint: **q** ≥ 0.0.

On entry, **r** = *<value>*.

Constraint: **r** ≥ 0.0.

On entry, **s** = *<value>*.

Constraint: **s** ≥ *<value>* and **s** ≤ *<value>*.

On entry, **sigma** = *<value>*.

Constraint: **sigma** > 0.0.

NE_REAL_ARRAY

On entry, $\mathbf{t}[\langle value \rangle] = \langle value \rangle$.

Constraint: $\mathbf{t}[i] \geq \langle value \rangle$.

On entry, $\mathbf{x}[\langle value \rangle] = \langle value \rangle$.

Constraint: $\mathbf{x}[i] \geq \langle value \rangle$ and $\mathbf{x}[i] \leq \langle value \rangle$.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ . This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the *machine precision* (see nag_cumul_normal (s15abc) and nag_erfc (s15adc)). An accuracy close to *machine precision* can generally be expected.

8 Parallelism and Performance

nag_binary_con_greeks (s30cbc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

None.

10 Example

This example computes the price of a cash-or-nothing call with a time to expiry of 0.75 years, a stock price of 110 and a strike price of 87. The risk-free interest rate is 5% per year, there is an annual dividend return of 4% and the volatility is 35% per year. If the option is in-the-money at expiration, i.e., if $S > X$, the payoff is 5.

10.1 Program Text

```

/* nag_binary_con_greeks (s30cbc) Example Program.
 *
 * Copyright 2009, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    /* Integer scalar and array declarations */
    Integer      exit_status = 0;
    Integer      i, j, m, n;
    NagError     fail;
    Nag_CallPut  putnum;
    /* Double scalar and array declarations */
    double       k, q, r, s, sigma;
    double       *charm = 0, *colour = 0, *crho = 0, *delta = 0, *gamma = 0;
    double       *p = 0, *rho = 0, *speed = 0, *t = 0, *theta = 0, *vanna = 0;
    double       *vega = 0, *vomma = 0, *x = 0, *zomma = 0;
    /* Character scalar and array declarations */
    char         put[8+1];
    Nag_OrderType order;

```

```

INIT_FAIL(fail);

printf("nag_binary_con_greeks (s30cbc) Example Program Results\n");
printf("Binary (Digital): Cash-or-Nothing\n\n");
/* Skip heading in data file */
scanf("%*[\n] ");
/* Read put */
scanf("%8s%*[\n] ", put);
/*
 * nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
putnum = (Nag_CallPut) nag_enum_name_to_value(put);
/* Read s, k, sigma, r, q */
scanf("%lf%lf%lf%lf%lf%*[\n] ", &s, &k, &sigma, &r, &q);
/* Read m, n */
scanf("%ld%ld%*[\n] ", &m, &n);
#ifdef NAG_COLUMN_MAJOR
#define CHARM(I, J)   charm[(J-1)*m + I-1]
#define COLOUR(I, J) colour[(J-1)*m + I-1]
#define CRHO(I, J)   crho[(J-1)*m + I-1]
#define DELTA(I, J)  delta[(J-1)*m + I-1]
#define GAMMA(I, J)  gamma[(J-1)*m + I-1]
#define P(I, J)       p[(J-1)*m + I-1]
#define RHO(I, J)     rho[(J-1)*m + I-1]
#define SPEED(I, J)  speed[(J-1)*m + I-1]
#define THETA(I, J)  theta[(J-1)*m + I-1]
#define VANNA(I, J)  vanna[(J-1)*m + I-1]
#define VEGA(I, J)   vega[(J-1)*m + I-1]
#define VOMMA(I, J)  vomma[(J-1)*m + I-1]
#define ZOMMA(I, J)  zomma[(J-1)*m + I-1]
order = Nag_ColMajor;
#else
#define CHARM(I, J)   charm[(I-1)*n + J-1]
#define COLOUR(I, J) colour[(I-1)*n + J-1]
#define CRHO(I, J)   crho[(I-1)*n + J-1]
#define DELTA(I, J)  delta[(I-1)*n + J-1]
#define GAMMA(I, J)  gamma[(I-1)*n + J-1]
#define P(I, J)       p[(I-1)*n + J-1]
#define RHO(I, J)     rho[(I-1)*n + J-1]
#define SPEED(I, J)  speed[(I-1)*n + J-1]
#define THETA(I, J)  theta[(I-1)*n + J-1]
#define VANNA(I, J)  vanna[(I-1)*n + J-1]
#define VEGA(I, J)   vega[(I-1)*n + J-1]
#define VOMMA(I, J)  vomma[(I-1)*n + J-1]
#define ZOMMA(I, J)  zomma[(I-1)*n + J-1]
order = Nag_RowMajor;
#endif
if (!(charm = NAG_ALLOC(m*n, double)) ||
    !(colour = NAG_ALLOC(m*n, double)) ||
    !(crho = NAG_ALLOC(m*n, double)) ||
    !(delta = NAG_ALLOC(m*n, double)) ||
    !(gamma = NAG_ALLOC(m*n, double)) ||
    !(p = NAG_ALLOC(m*n, double)) ||
    !(rho = NAG_ALLOC(m*n, double)) ||
    !(speed = NAG_ALLOC(m*n, double)) ||
    !(t = NAG_ALLOC(n, double)) ||
    !(theta = NAG_ALLOC(m*n, double)) ||
    !(vanna = NAG_ALLOC(m*n, double)) ||
    !(vega = NAG_ALLOC(m*n, double)) ||
    !(vomma = NAG_ALLOC(m*n, double)) ||
    !(x = NAG_ALLOC(m, double)) ||
    !(zomma = NAG_ALLOC(m*n, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
/* Read array of strike/exercise prices, X */
for (i = 0; i < m; i++)

```

```

    scanf("%lf ", &x[i]);
    scanf("%*[^\\n] ");
    /* Read array of times to expiry */
    for (i = 0; i < n; i++)
        scanf("%lf ", &t[i]);
    scanf("%*[^\\n] ");
    /*
    * nag_binary_con_greeks (s30cbc)
    * Binary option: Cash-or-nothing pricing formula with Greeks
    */
    nag_binary_con_greeks(order, putnum, m, n, x, s, k, t, sigma, r, q, p,
                        delta, gamma, vega, theta, rho, crho, vanna, charm,
                        speed, colour, zomma, vomma, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_binary_con_greeks (s30cbc).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }
    if (putnum == Nag_Call)
        printf("European Call :\n\n");
    else if (putnum == Nag_Put)
        printf("European Put :\n\n");
    printf("%s%8.4f\n", " Spot      = ", s);
    printf("%s%8.4f\n", " Payout    = ", k);
    printf("%s%8.4f\n", " Volatility = ", sigma);
    printf("%s%8.4f\n", " Rate     = ", r);
    printf("%s%8.4f\n", " Dividend = ", q);
    printf("\n");
    for (j = 1; j <= n; j++)
    {
        printf("\n");
        printf(" Time to Expiry : %8.4f\n", t[j-1]);
        printf(" Strike Price Delta Gamma Vega Theta "
              " Rho CRho\n");
        for (i = 1; i <= m; i++)
        {
            printf("%8.4f%8.4f%8.4f%8.4f%8.4f%8.4f%8.4f \n",
                  x[i-1], P(i, j), DELTA(i, j), GAMMA(i, j), VEGA(i, j),
                  THETA(i, j), RHO(i, j), CRHO(i, j));
        }
        printf("          Vanna Charm Speed Colour "
              "Zomma Vomma\n");
        for (i = 1; i <= m; i++)
        {
            printf("%24.4f%8.4f%8.4f%8.4f%8.4f%8.4f \n",
                  VANNA(i, j), CHARM(i, j), SPEED(i, j), COLOUR(i, j),
                  ZOMMA(i, j), VOMMA(i, j));
        }
    }
}

END:
NAG_FREE(charm);
NAG_FREE(colour);
NAG_FREE(crho);
NAG_FREE(delta);
NAG_FREE(gamma);
NAG_FREE(p);
NAG_FREE(rho);
NAG_FREE(speed);
NAG_FREE(t);
NAG_FREE(theta);
NAG_FREE(vanna);
NAG_FREE(vega);
NAG_FREE(vomma);
NAG_FREE(x);
NAG_FREE(zomma);

return exit_status;
}

```


10.2 Program Data

```
nag_binary_con_greeks (s30cbc) Example Program Data
Nag_Call           : Nag_Call or Nag_Put
110.0 5.0 0.35 0.05 0.04 : s, k, sigma, r, q
1 1                : m, n
87.0               : X(I), I = 1,2,...m
0.75               : T(I), I = 1,2,...n
```

10.3 Program Results

```
nag_binary_con_greeks (s30cbc) Example Program Results
Binary (Digital): Cash-or-Nothing
```

European Call :

```
Spot      = 110.0000
Payout    = 5.0000
Volatility = 0.3500
Rate      = 0.0500
Dividend  = 0.0400
```

```
Time to Expiry : 0.7500
Strike Price   Delta Gamma Vega Theta Rho CRho
87.0000 3.5696 0.0467 -0.0013 -4.2307 1.1142 1.1788 3.8560
          Vanna Charm Speed Colour Zomma Vomma
          -0.0514 0.0153 0.0000 -0.0019 0.0079 12.8874
```
