

NAG Library Function Document

nag_elliptic_integral_complete_K (s21bhc)

1 Purpose

nag_elliptic_integral_complete_K (s21bhc) returns a value of the classical (Legendre) form of the complete elliptic integral of the first kind.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_elliptic_integral_complete_K (double dm, NagError *fail)
```

3 Description

nag_elliptic_integral_complete_K (s21bhc) calculates an approximation to the integral

$$K(m) = \int_0^{\frac{\pi}{2}} (1 - m \sin^2 \theta)^{-\frac{1}{2}} d\theta,$$

where $m < 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$K(m) = R_F(0, 1 - m, 1),$$

where R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see nag_elliptic_integral_rf (s21bbc)).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Arguments

1: **dm** – double *Input*

On entry: the argument m of the function.

Constraint: **dm** < 1.0.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **dm** = $\langle value \rangle$; the integral is undefined.
 Constraint: **dm** < 1.0.
 On failure, the function returns zero.

NW_INTEGRAL_INFINITY

On entry, **dm** = 1.0; the integral is infinite.
 On failure, the function returns the largest machine number (see `nag_real_largest_number` (X02ALC)).

7 Accuracy

In principle `nag_elliptic_integral_complete_K` (s21bhc) is capable of producing full *machine precision*. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the *machine precision*.

8 Parallelism and Performance

Not applicable.

9 Further Comments

You should consult the s Chapter Introduction, which shows the relationship between this function and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithm used to compute R_F , see the function document for `nag_elliptic_integral_rf` (s21bbc).

10 Example

This example simply generates a small set of nonextreme arguments that are used with the function to produce the table of results.

10.1 Program Text

```

/* nag_elliptic_integral_complete_K (s21bhc) Example Program.
 *
 * Copyright 2008, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  /*Integer scalar and array declarations */
  Integer  exit_status = 0;
  Integer  ix;
  /*Double scalar and array declarations */
  double  dm, k;
  NagError fail;

  INIT_FAIL(fail);

  printf("%s\n",

```

```

        "nag_elliptic_integral_complete_K (s21bhc) Example Program Results");
printf("\n      dm      nag_elliptic_integral_complete_K\n\n");
for (ix = 1; ix <= 3; ix++)
{
    dm = ix*0.250e0;
    /*
     * nag_elliptic_integral_complete_K (s21bhc)
     * Complete elliptic integral of 1st kind, Legendre form, K(m)
     */
    k = nag_elliptic_integral_complete_K(dm, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from "
              "nag_elliptic_integral_complete_K (s21bhc).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }
    printf("%7.2f%12.4f\n", dm, k);
}

END:

    return exit_status;
}

```

10.2 Program Data

None.

10.3 Program Results

nag_elliptic_integral_complete_K (s21bhc) Example Program Results

dm	nag_elliptic_integral_complete_K
0.25	1.6858
0.50	1.8541
0.75	2.1565
