NAG Library Function Document

nag_bessel_i0 (s18aec)

1 Purpose

nag_bessel_i0 (s18aec) returns the value of the modified Bessel function $I_0(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_bessel_i0 (double x, NagError *fail)
```

3 Description

nag_bessel_i0 (s18aec) evaluates an approximation to the modified Bessel function of the first kind $I_0(x)$.

Note: $I_0(-x) = I_0(x)$, so the approximation need only consider $x \geq 0$.

The function is based on three Chebyshev expansions:

For $0 < x \leq 4$,

$$I_0(x) = e^x \sum_{r=0} a_r T_r(t), \qquad \text{where } t = 2\left(\frac{x}{4}\right) - 1.$$

For $4 < x \leq 12$,

$$I_0(x) = e^x \sum_{r=0} b_r T_r(t), \qquad \text{where } t = \frac{x-8}{4}.$$

For $x > 12$,

$$I_0(x) = \frac{e^x}{\sqrt{x}} \sum_{r=0} c_r T_r(t), \qquad \text{where } t = 2\left(\frac{12}{x}\right) - 1.$$

For small x, $I_0(x) \simeq 1$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For large x, the function must fail because of the danger of overflow in calculating e^x.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GT

On entry, **x** = ⟨*value*⟩.
Constraint: |**x**| ≤ ⟨*value*⟩.
|**x**| is too large and the function returns the approximate value of $I_0(x)$ at the nearest valid argument.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the ***machine precision*** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left|\frac{xI_1(x)}{I_0(x)}\right|\delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left|\frac{xI_1(x)}{I_0(x)}\right|.$$

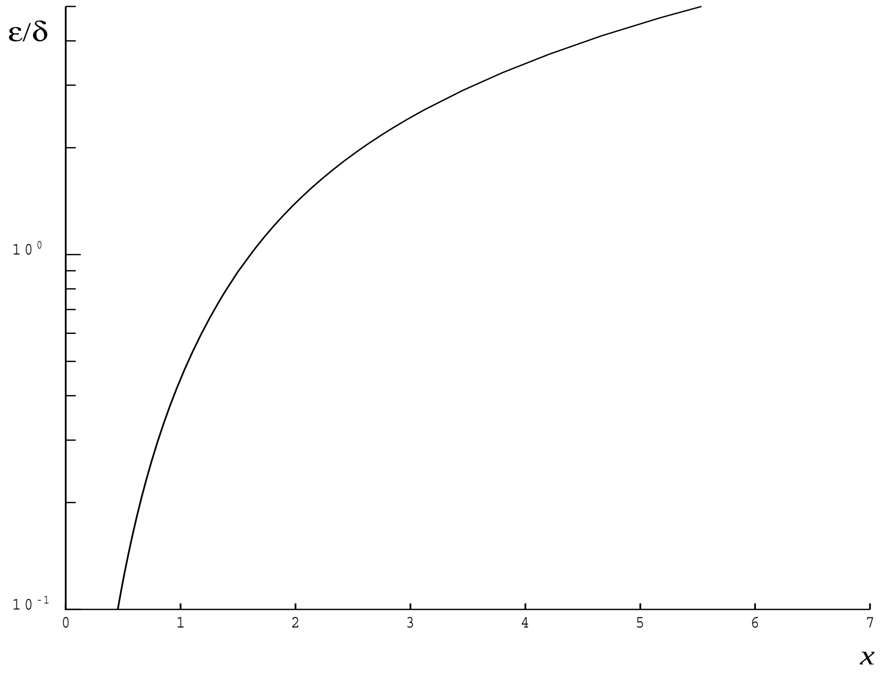


Figure 1

However if δ is of the same order as ***machine precision***, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x the amplification factor is approximately $\frac{x^2}{2}$, which implies strong attenuation of the error, but in general ϵ can never be less than the ***machine precision***.

For large x, $\epsilon \simeq x\delta$ and we have strong amplification of errors. However the function must fail for quite moderate values of x, because $I_0(x)$ would overflow; hence in practice the loss of accuracy for large x is not excessive. Note that for large x the errors will be dominated by those of the standard function exp.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

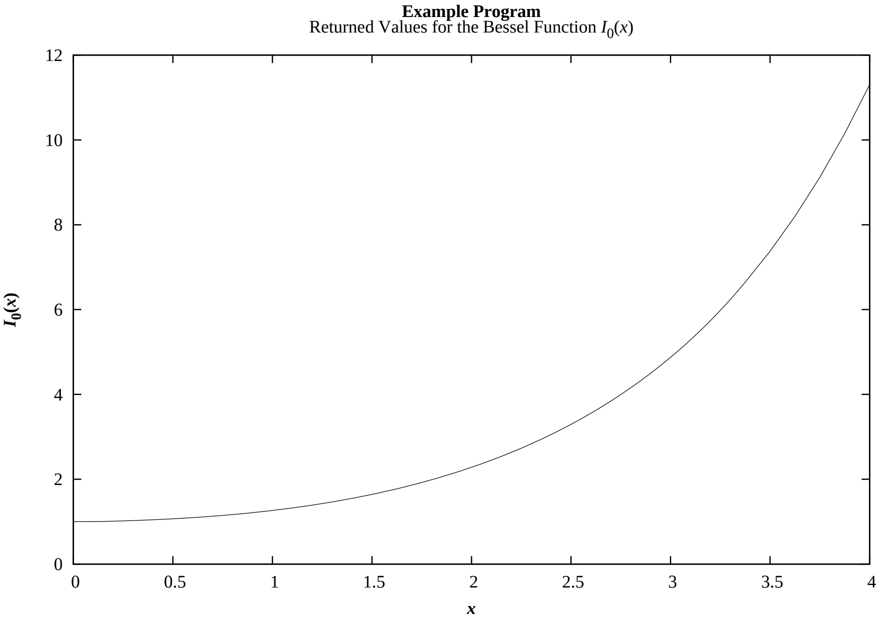
```
/* nag_bessel_i0 (s18aec) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_bessel_i0 (s18aec) Example Program Results\n");
  printf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_bessel_i0 (s18aec).
       * Modified Bessel function I_0(x)
       */
      y = nag_bessel_i0(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_bessel_i0 (s18aec).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_bessel_i0 (s18aec) Example Program Data
             0.0
             0.5
             1.0
             3.0
             6.0
             8.0
             10.0
             15.0
             20.0
             -1.0
```

10.3 Program Results

```
nag_bessel_i0 (s18aec) Example Program Results
     x           y
   0.000e+00   1.000e+00
   5.000e-01   1.063e+00
   1.000e+00   1.266e+00
   3.000e+00   4.881e+00
   6.000e+00   6.723e+01
   8.000e+00   4.276e+02
   1.000e+01   2.816e+03
   1.500e+01   3.396e+05
   2.000e+01   4.356e+07
  -1.000e+00   1.266e+00
```


Example Program
Returned Values for the Bessel Function $I_0(x)$
$I_0(x)$
x