NAG Library Function Document

nag_bessel_k1 (s18adc)

1 Purpose

nag_bessel_k1 (s18adc) returns the value of the modified Bessel function $K_1(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_bessel_k1 (double x, NagError *fail)
```

3 Description

nag_bessel_k1 (s18adc) evaluates an approximation to the modified Bessel function of the second kind $K_1(x)$.

Note: $K_1(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on five Chebyshev expansions:

For $0 < x \leq 1$,

$$K_1(x) = \frac{1}{x} + x \ln x \sum_{r=0} a_r T_r(t) - x \sum_{r=0} b_r T_r(t), \qquad \text{where } t = 2x^2 - 1.$$

For $1 < x \leq 2$,

$$K_1(x) = e^{-x} \sum_{r=0} c_r T_r(t), \qquad \text{where } t = 2x - 3.$$

For $2 < x \leq 4$,

$$K_1(x) = e^{-x} \sum_{r=0} d_r T_r(t), \qquad \text{where } t = x - 3.$$

For $x > 4$,

$$K_1(x) = \frac{e^{-x}}{\sqrt{x}} \sum_{r=0} e_r T_r(t), \qquad \text{where } t = \frac{9 - x}{1 + x}.$$

For x near zero, $K_1(x) \simeq \frac{1}{x}$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***. For very small x on some machines, it is impossible to calculate $\frac{1}{x}$ without overflow and the function must fail.

For large x, where there is a danger of underflow due to the smallness of K_1, the result is set exactly to zero.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: **x** > 0.0.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_LE

On entry, **x** $= \langle value \rangle$.
Constraint: **x** > 0.0.
K_0 is undefined and the function returns zero.

NE_REAL_ARG_TOO_SMALL

On entry, **x** $= \langle value \rangle$.
Constraint: **x** $> \langle value \rangle$.
x is too small, there is a danger of overflow and the function returns approximately the largest representable value.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the ***machine precision*** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{xK_0(x) - K_1(x)}{K_1(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left| \frac{xK_0(x) - K_1(x)}{K_1(x)} \right|.$$

However if δ is of the same order as the ***machine precision***, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x, $\epsilon \simeq \delta$ and there is no amplification of errors.

For large x, $\epsilon \simeq x\delta$ and we have strong amplification of the relative error. Eventually K_1, which is asymptotically given by $\frac{e^{-x}}{\sqrt{x}}$, becomes so small that it cannot be calculated without underflow and hence the function will return zero. Note that for large x the errors will be dominated by those of the standard function exp.

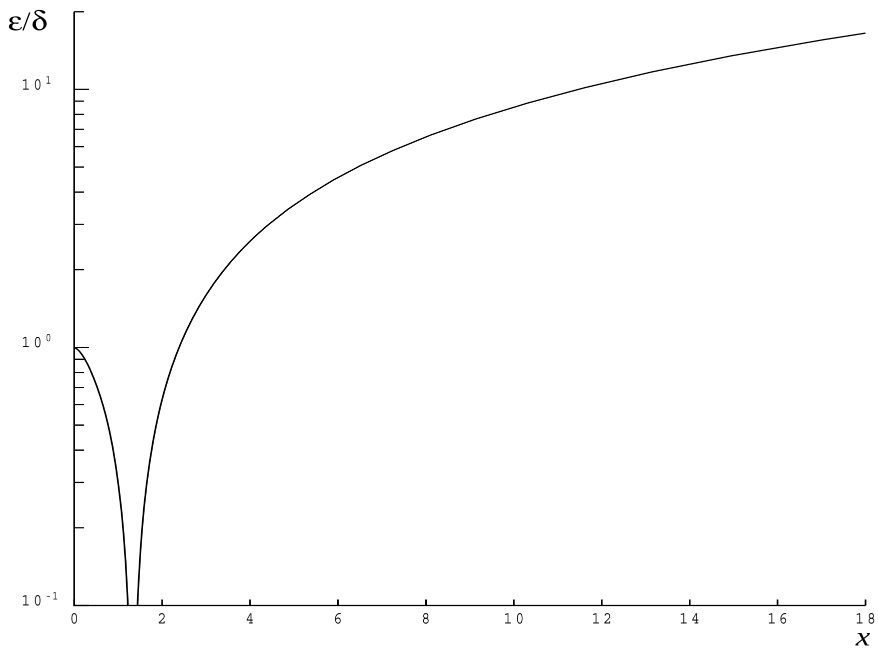


Figure 1

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_bessel_k1 (s18adc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_bessel_k1 (s18adc) Example Program Results\n");
  printf("     x           y\n");
```

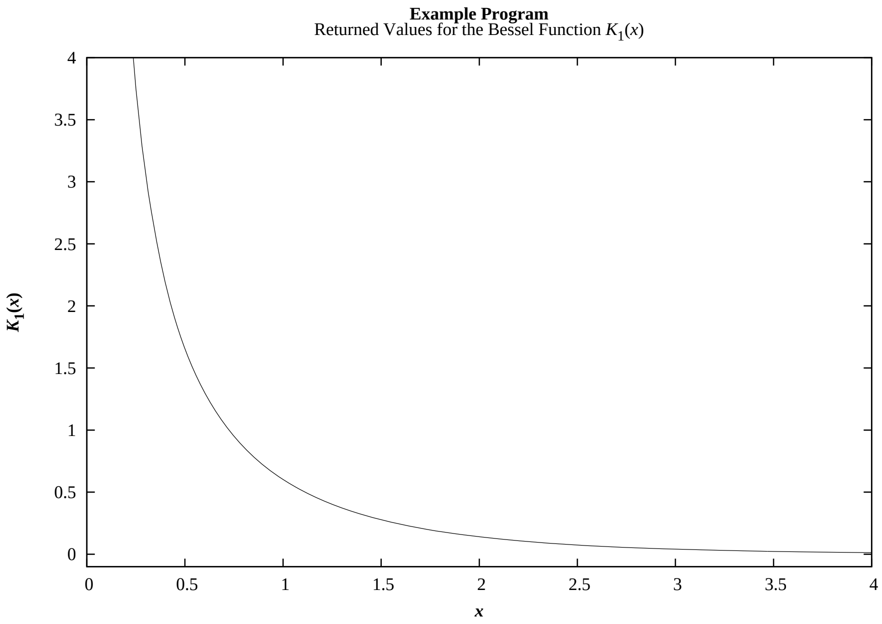
```
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_bessel_k1 (s18adc).
       * Modified Bessel function K_1(x)
       */
      y = nag_bessel_k1(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_bessel_k1 (s18adc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_bessel_k1 (s18adc) Example Program Data
             0.4
             0.6
             1.4
             1.6
             2.5
             3.5
             6.0
             8.0
             10.0
             1000.0
```

10.3 Program Results

```
nag_bessel_k1 (s18adc) Example Program Results
     x           y
   4.000e-01   2.184e+00
   6.000e-01   1.303e+00
   1.400e+00   3.208e-01
   1.600e+00   2.406e-01
   2.500e+00   7.389e-02
   3.500e+00   2.224e-02
   6.000e+00   1.344e-03
   8.000e+00   1.554e-04
   1.000e+01   1.865e-05
   1.000e+03   0.000e+00
```


Example Program
Returned Values for the Bessel Function $K_1(x)$
$K_1(x)$
x
0
0.5
1
1.5
2
2.5
3
3.5
4
