# NAG Library Function Document

# nag_bessel_k0 (s18acc)

## 1 Purpose

nag_bessel_k0 (s18acc) returns the value of the modified Bessel function $K_0(x)$.

## 2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_bessel_k0 (double x, NagError *fail)
```

## 3 Description

nag_bessel_k0 (s18acc) evaluates an approximation to the modified Bessel function of the second kind $K_0(x)$.

**Note:** $K_0(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on five Chebyshev expansions:

For $0 < x \leq 1$,

$$K_0(x) = -\ln x \sum_{r=0} a_r T_r(t) + \sum_{r=0} b_r T_r(t), \qquad \text{where } t = 2x^2 - 1.$$

For $1 < x \leq 2$,

$$K_0(x) = e^{-x} \sum_{r=0} c_r T_r(t), \qquad \text{where } t = 2x - 3.$$

For $2 < x \leq 4$,

$$K_0(x) = e^{-x} \sum_{r=0} d_r T_r(t), \qquad \text{where } t = x - 3.$$

For $x > 4$,

$$K_0(x) = \frac{e^{-x}}{\sqrt{x}} \sum_{r=0} e_r T_r(t), \text{where } t = \frac{9-x}{1+x}.$$

For $x$ near zero, $K_0(x) \simeq -\gamma - \ln\left(\frac{x}{2}\right)$, where $\gamma$ denotes Euler's constant. This approximation is used when $x$ is sufficiently small for the result to be correct to ***machine precision***.

For large $x$, where there is a danger of underflow due to the smallness of $K_0$, the result is set exactly to zero.

## 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

# 5 Arguments

1: **x** – double *Input*

*On entry*: the argument $x$ of the function.

*Constraint*: $\mathbf{x} > 0.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

# 6 Error Indicators and Warnings

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE_REAL_ARG_LE**

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} > 0.0$.
$K_0$ is undefined and the function returns zero.

# 7 Accuracy

Let $\delta$ and $\epsilon$ be the relative errors in the argument and result respectively.

If $\delta$ is somewhat larger than the ***machine precision*** (i.e., if $\delta$ is due to data errors etc.), then $\epsilon$ and $\delta$ are approximately related by:

$$\epsilon \simeq \left| \frac{xK_1(x)}{K_0(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left| \frac{xK_1(x)}{K_0(x)} \right|.$$

However, if $\delta$ is of the same order as ***machine precision***, then rounding errors could make $\epsilon$ slightly larger than the above relation predicts.

For small $x$, the amplification factor is approximately $\left| \frac{1}{\ln x} \right|$, which implies strong attenuation of the error, but in general $\epsilon$ can never be less than the ***machine precision***.

For large $x$, $\epsilon \simeq x\delta$ and we have strong amplification of the relative error. Eventually $K_0$, which is asymptotically given by $\frac{e^{-x}}{\sqrt{x}}$, becomes so small that it cannot be calculated without underflow and hence the function will return zero. Note that for large $x$ the errors will be dominated by those of the standard math library function exp.

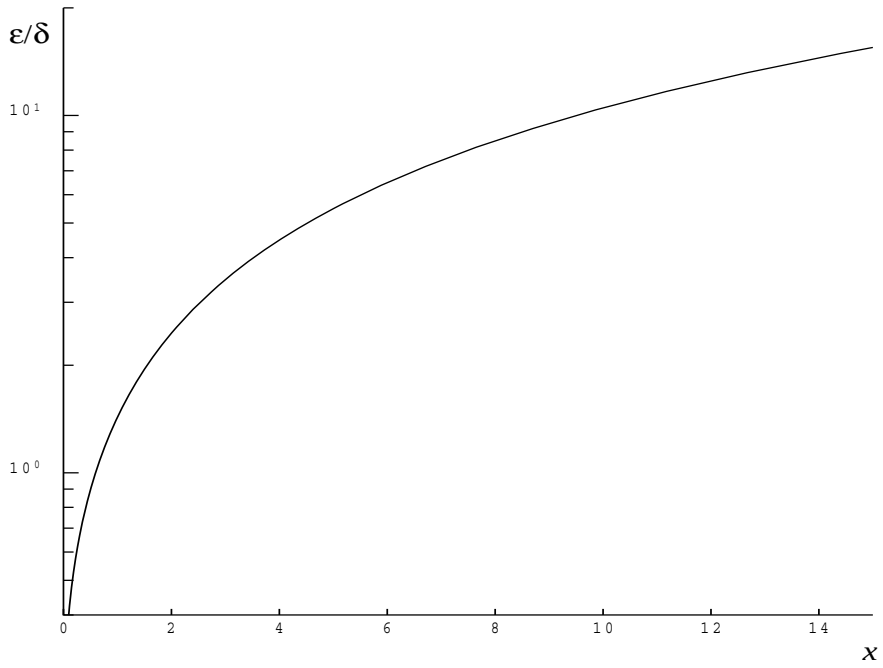



**Figure 1**

## 8 Parallelism and Performance

Not applicable.

## 9 Further Comments

None.

## 10 Example

This example reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

### 10.1 Program Text

```
/* nag_bessel_k0 (s18acc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_bessel_k0 (s18acc) Example Program Results\n");
```

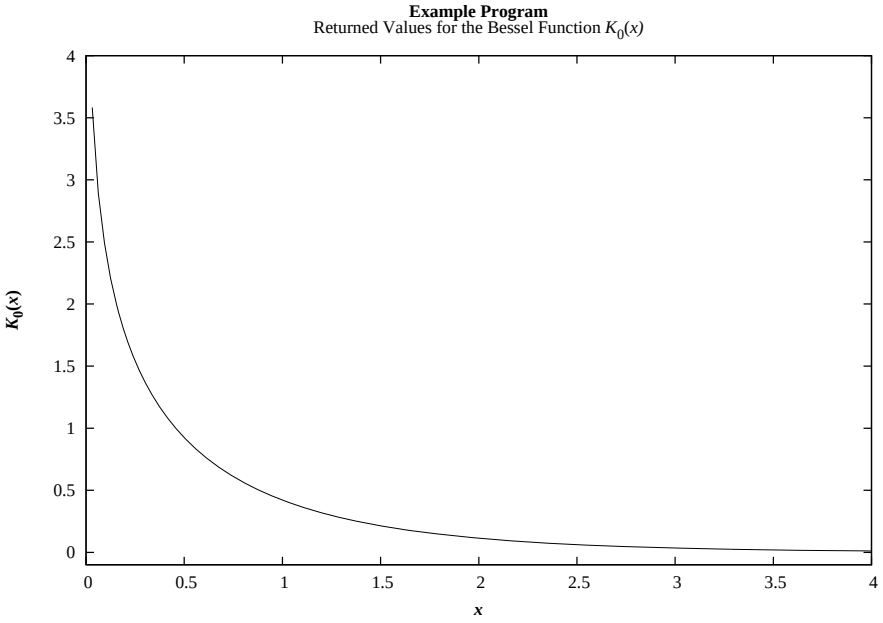
```
  printf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_bessel_k0 (s18acc).
       * Modified Bessel function K_0(x)
       */
      y = nag_bessel_k0(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_bessel_k0 (s18acc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

## 10.2 Program Data

```
nag_bessel_k0 (s18acc) Example Program Data
             0.4
             0.6
             1.4
             1.6
             2.5
             3.5
             6.0
             8.0
             10.0
             1000.0
```

## 10.3 Program Results

```
nag_bessel_k0 (s18acc) Example Program Results
     x           y
   4.000e-01   1.115e+00
   6.000e-01   7.775e-01
   1.400e+00   2.437e-01
   1.600e+00   1.880e-01
   2.500e+00   6.235e-02
   3.500e+00   1.960e-02
   6.000e+00   1.244e-03
   8.000e+00   1.465e-04
   1.000e+01   1.778e-05
   1.000e+03   0.000e+00
```


Example Program
Returned Values for the Bessel Function $K_0(x)$
$K_0(x)$
$x$
0
0.5
1
1.5
2
2.5
3
3.5
4