NAG Library Function Document

nag_complex_polygamma (s14afc)

1 Purpose

nag_complex_polygamma (s14afc) returns the value of the kth derivative of the psi function $\psi(z)$ for complex z and $k = 0, 1, \ldots, 4$.

2 Specification

```
#include <nag.h>
#include <nags.h>
Complex nag_complex_polygamma (Complex z, Integer k, NagError *fail)
```

3 Description

nag_complex_polygamma (s14afc) evaluates an approximation to the kth derivative of the psi function $\psi(z)$ given by

$$\psi^{(k)}(z) = \frac{d^k}{dz^k}\psi(z) = \frac{d^k}{dz^k}\left(\frac{d}{dz}\log_{\mathrm{e}}\Gamma(z)\right),$$

where $z = x + iy$ is complex provided $y \neq 0$ and $k = 0, 1, \ldots, 4$. If $y = 0$, z is real and thus $\psi^{(k)}(z)$ is singular when $z = 0, -1, -2, \ldots$.

Note that $\psi^{(k)}(z)$ is also known as the *polygamma* function. Specifically, $\psi^{(0)}(z)$ is often referred to as the *digamma* function and $\psi^{(1)}(z)$ as the *trigamma* function in the literature. Further details can be found in Abramowitz and Stegun (1972).

nag_complex_polygamma (s14afc) is based on a modification of the method proposed by Kölbig (1972).

To obtain the value of $\psi^{(k)}(z)$ when z is real, nag_real_polygamma (s14aec) can be used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Kölbig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

5 Arguments

1: **z** – Complex *Input*

On entry: the argument z of the function.

Constraint: **z**.re must not be 'too close' (see Section 6) to a non-positive integer when **z**.$im = 0.0$.

2: **k** – Integer *Input*

On entry: the function $\psi^{(k)}(z)$ to be evaluated.

Constraint: $0 \leq \mathbf{k} \leq 4$.

3: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_COMPLEX

On entry, $\mathbf{z}.re$ is 'too close' to a non-positive integer when $\mathbf{z}.im = 0.0$: $\mathbf{z}.re = \langle value \rangle$, nint($\mathbf{z}.re$) = $\langle value \rangle$.

NE_INT

On entry, $\mathbf{k} = \langle value \rangle$.
Constraint: $\mathbf{k} \leq 4$.

On entry, $\mathbf{k} = \langle value \rangle$.
Constraint: $\mathbf{k} \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_OVERFLOW_LIKELY

Evaluation abandoned due to likelihood of overflow.

7 Accuracy

Empirical tests have shown that the maximum relative error is a loss of approximately two decimal places of precision.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example evaluates the psi (trigamma) function $\psi^{(1)}(z)$ at $z = -1.5 + 2.5i$, and prints the results.

10.1 Program Text

```
/* nag_complex_polygamma (s14afc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Complex  y, z;
  Integer  exit_status = 0, k;
  NagError fail;
```

```
  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n] ");
  printf("nag_complex_polygamma (s14afc) Example Program Results\n");
  printf("       z           k         (d^k/dz^k)psi(z)\n");
  while (scanf(" (%lf,%lf)%ld%*[^\n] ", &z.re, &z.im, &k) != EOF)
    {
      /* nag_complex_polygamma (s14afc).
       * Derivative of the psi function psi(z)
       */
      y = nag_complex_polygamma(z, k, &fail);
      if (fail.code == NE_NOERROR)
        printf("(%5.1f, %5.1f) %6ld (%13.4e, %13.4e)\n",
                z.re, z.im, k, y.re, y.im);
      else
        {
          printf("Error from nag_complex_polygamma (s14afc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
    }
 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_complex_polygamma (s14afc) Example Program Data
(1.2,5.0)   0
(0.5,-0.2)  1
(-1.5,2.5)  1
(8.0,3.3)   3
(2.9,7.5)   4  : Values of z and k
```

10.3 Program Results

```
nag_complex_polygamma (s14afc) Example Program Results
       z            k         (d^k/dz^k)psi(z)
(  1.2,   5.0)      0 (  1.6176e+00,   1.4312e+00)
(  0.5,  -0.2)      1 (  3.4044e+00,   2.5394e+00)
( -1.5,   2.5)      1 ( -1.9737e-01,  -2.4271e-01)
(  8.0,   3.3)      3 (  1.1814e-03,  -3.4188e-03)
(  2.9,   7.5)      4 ( -5.0227e-04,  -1.4955e-03)
```