NAG Library Function Document

nag_arctanh (s11aac)

1 Purpose

nag_arctanh (s11aac) returns the value of the inverse hyperbolic tangent, $\operatorname{arctanh} x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_arctanh (double x, NagError *fail)
```

3 Description

nag_arctanh (s11aac) calculates an approximate value for the inverse hyperbolic tangent of its argument, $\operatorname{arctanh} x$.

For $x^2 \leq \frac{1}{2}$ it is based on the Chebyshev expansion

$$\operatorname{arctanh} x = x \times y(t) = x\sum_{r=0} a_r T_r(t)$$

where $-\dfrac{1}{\sqrt{2}} \leq x \leq \dfrac{1}{\sqrt{2}}, \quad -1 \leq t \leq 1, \quad$ and $\quad t = 4x^2 - 1.$

For $\frac{1}{2} < x^2 < 1$, it uses

$$\operatorname{arctanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right).$$

For $|x| \geq 1$, the function fails as $\operatorname{arctanh} x$ is undefined.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $|\mathbf{x}| < 1.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GE

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $|\mathbf{x}| < 1$.
The function has been called with an argument greater than or equal to 1.0 in magnitude, for which arctanh is not defined.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{(1-x^2)\operatorname{arctanh} x} \times \delta \right|.$$

That is, the relative error in the argument, x, is amplified by at least a factor $\dfrac{x}{(1-x^2)\operatorname{arctanh} x}$ in the result. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

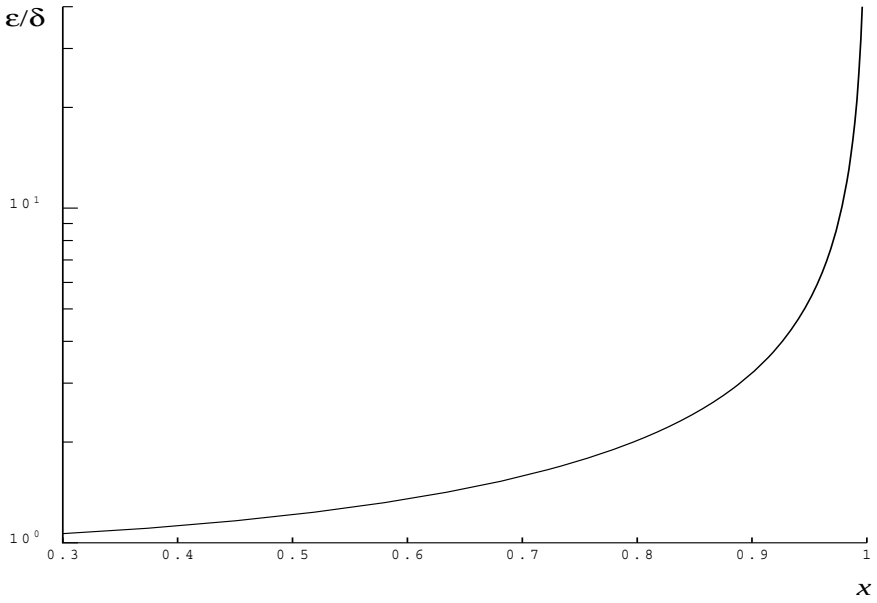

Figure 1

The factor is not significantly greater than one except for arguments close to $|x| = 1$. However in the region where $|x|$ is close to one, $1 - |x| \sim \delta$, the above analysis is inapplicable since x is bounded by definition, $|x| < 1$. In this region where arctanh is tending to infinity we have

$$\epsilon \sim 1/\ln \delta$$

which implies an obvious, unavoidable serious loss of accuracy near $|x| \sim 1$, e.g., if x and 1 agree to 6 significant figures, the result for $\operatorname{arctanh} x$ would be correct to at most about one figure.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_arctanh (s11aac) Example Program.
 *
 * Copyright 1989 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_arctanh (s11aac) Example Program Results\n");
  printf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_arctanh (s11aac).
       * Inverse hyperbolic tangent, arctanh x
       */
      y = nag_arctanh(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_arctanh (s11aac).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_arctanh (s11aac) Example Program Data
      -0.5
       0.0
       0.5
      -0.9999
```

10.3 Program Results

```
nag_arctanh (s11aac) Example Program Results
     x           y
  -5.000e-01  -5.493e-01
   0.000e+00   0.000e+00
   5.000e-01   5.493e-01
  -9.999e-01  -4.952e+00
```