

NAG Library Function Document

nag_nearest_correlation (g02aac)

1 Purpose

nag_nearest_correlation (g02aac) computes the nearest correlation matrix, in the Frobenius norm, to a given square, input matrix.

2 Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_nearest_correlation (Nag_OrderType order, double g[], Integer pdg,
    Integer n, double errtol, Integer maxits, Integer maxit, double x[],
    Integer pdx, Integer *iter, Integer *feval, double *nrmgrd,
    NagError *fail)
```

3 Description

A correlation matrix may be characterised as a real square matrix that is symmetric, has a unit diagonal and is positive semidefinite.

nag_nearest_correlation (g02aac) applies an inexact Newton method to a dual formulation of the problem, as described by Qi and Sun (2006). It applies the improvements suggested by Borsdorf and Higham (2010).

4 References

Borsdorf R and Higham N J (2010) A preconditioned (Newton) algorithm for the nearest correlation matrix *IMA Journal of Numerical Analysis* **30(1)** 94–107

Qi H and Sun D (2006) A quadratically convergent Newton method for computing the nearest correlation matrix *SIAM J. Matrix AnalAppl* **29(2)** 360–385

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **g**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **g** must be at least **pdg** × **n**.

The (*i*, *j*)th element of the matrix *G* is stored in

$$\begin{aligned} & \mathbf{g}[(j-1) \times \mathbf{pdg} + i - 1] \text{ when } \mathbf{order} = \text{Nag_ColMajor}; \\ & \mathbf{g}[(i-1) \times \mathbf{pdg} + j - 1] \text{ when } \mathbf{order} = \text{Nag_RowMajor}. \end{aligned}$$

On entry: *G*, the initial matrix.

On exit: a symmetric matrix $\frac{1}{2}(G + G^T)$ with the diagonal set to *I*.

- 3: **pdg** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) in the array **g**.
Constraint: **pdg** \geq **n**.
- 4: **n** – Integer *Input*
On entry: the size of the matrix *G*.
Constraint: **n** $>$ 0.
- 5: **errtol** – double *Input*
On entry: the termination tolerance for the Newton iteration. If **errtol** \leq 0.0 then **n** \times $\sqrt{\text{machine precision}}$ is used.
- 6: **maxits** – Integer *Input*
On entry: **maxits** specifies the maximum number of iterations used for the iterative scheme used to solve the linear algebraic equations at each Newton step.
 If **maxits** \leq 0, $2 \times \mathbf{n}$ is used.
- 7: **maxit** – Integer *Input*
On entry: specifies the maximum number of Newton iterations.
 If **maxit** \leq 0, 200 is used.
- 8: **x**[*dim*] – double *Output*
Note: the dimension, *dim*, of the array **x** must be at least **pdx** \times **n**.
 The (*i*, *j*)th element of the matrix *X* is stored in
 x[(*j* – 1) \times **pdx** + *i* – 1] when **order** = Nag_ColMajor;
 x[(*i* – 1) \times **pdx** + *j* – 1] when **order** = Nag_RowMajor.
On exit: contains the nearest correlation matrix.
- 9: **pdx** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) in the array **x**.
Constraint: **pdx** \geq **n**.
- 10: **iter** – Integer * *Output*
On exit: the number of Newton steps taken.
- 11: **feval** – Integer * *Output*
On exit: the number of function evaluations of the dual problem.
- 12: **nrmgrd** – double * *Output*
On exit: the norm of the gradient of the last Newton step.
- 13: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_CONVERGENCE

Machine precision is limiting convergence.
The array returned in \mathbf{x} may still be of interest.
Newton iteration fails to converge in $\langle value \rangle$ iterations.

NE_EIGENPROBLEM

Failure to solve intermediate eigenproblem.

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} > 0$.

NE_INT_2

On entry, $\mathbf{pdg} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{pdg} \geq \mathbf{n}$.
On entry, $\mathbf{pdx} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{pdx} \geq \mathbf{n}$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

The returned accuracy is controlled by **errtol** and limited by *machine precision*.

8 Parallelism and Performance

`nag_nearest_correlation` (g02aac) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_nearest_correlation` (g02aac) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Arrays are internally allocated by `nag_nearest_correlation` (g02aac). The total size of these arrays is $11 \times \mathbf{n} + 3 \times \mathbf{n} \times \mathbf{n} + \max(2 \times \mathbf{n} \times \mathbf{n} + 6 \times \mathbf{n} + 1, 120 + 9 \times \mathbf{n})$ real elements and $5 \times \mathbf{n} + 3$ integer elements.

10 Example

This example finds the nearest correlation matrix to:

$$G = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

10.1 Program Text

```

/* nag_nearest_correlation (g02aac) Example Program.
 *
 * Copyright 2008, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg02.h>

int main(void)
{
    /*Integer scalar and array declarations */
    Integer      exit_status = 0;
    Integer      feval, i, iter, j, maxit, maxits, n;
    Integer      ldg, pdg, pdx;
    /*Double scalar and array declarations */
    double       errtol, nrmgrd;
    double       *g = 0, *x = 0;
    Nag_OrderType order;
    NagError     fail;

    INIT_FAIL(fail);

    printf("%s\n",
           "nag_nearest_correlation (g02aac) Example Program Results");
    printf("\n");
    n = 4;
    ldg = 5;
    #ifdef NAG_COLUMN_MAJOR
    pdg = ldg;
    #define G(I, J) g[(J-1)*pdg + I-1]
    pdx = n;
    #define X(I, J) x[(J-1)*pdx + I-1]
    order = Nag_ColMajor;
    #else
    pdg = n;
    #define G(I, J) g[(I-1)*pdg + J-1]
    pdx = n;
    #define X(I, J) x[(I-1)*pdx + J-1]
    order = Nag_RowMajor;
    #endif
    if (!(g = NAG_ALLOC(ldg*n, double)) ||
        !(x = NAG_ALLOC(n*n, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Set up matrix G*/
    for (j = 1; j <= n; j++)
    {
        for (i = 1; i <= n; i++)

```

```

        G(i, j) = 0.0;
        G(j, j) = 2.00e0;
    }
    for (j = 2; j <= n; j++)
    {
        G(j-1, j) = -(1.00e0);
        G(j, j-1) = -(1.00e0);
    }
    /* Set up method parameters*/
    errtol = 1.00e-7;
    maxits = 200;
    maxit = 10;
    /*
    * nag_nearest_correlation (g02aac)
    * Computes the nearest correlation matrix to a real square matrix,
    * using the method of Qi and Sun
    */
    nag_nearest_correlation(order, g, pdg, n, errtol, maxits, maxit, x, pdx,
        &iter, &feval, &nrmgrd, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_nearest_correlation (g02aac).\n%s\n",
            fail.message);
        exit_status = 1;
        goto END;
    }
    printf("%s\n", "      Nearest Correlation Matrix");
    for (i = 1; i <= n; i++)
    {
        for (j = 1; j <= n; j++)
            printf("%11.5f%s", X(i, j), j%4?" ":"\n");
    }
    printf("\n");
    printf("\n");
    printf("%s%11ld\n", " Number of Newton steps taken:", iter);
    printf("%s%9ld\n", " Number of function evaluations:", feval);
    if (nrmgrd > errtol)
        printf("%s %12.3e\n", " Norm of gradient of last Newton step:",
            nrmgrd);
    printf("\n");

END:
    NAG_FREE(g);
    NAG_FREE(x);

    return exit_status;
}

```

10.2 Program Data

None.

10.3 Program Results

nag_nearest_correlation (g02aac) Example Program Results

```

      Nearest Correlation Matrix
  1.00000   -0.80841    0.19159    0.10678
 -0.80841    1.00000   -0.65623    0.19159
  0.19159   -0.65623    1.00000   -0.80841
  0.10678    0.19159   -0.80841    1.00000

```

```

Number of Newton steps taken:      3
Number of function evaluations:    4

```
