NAG Library Function Document

nag_prob_non_central_chi_sq (g01gcc)

1 Purpose

nag_prob_non_central_chi_sq (g01gcc) returns the probability associated with the lower tail of the noncentral χ^2-distribution .

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_prob_non_central_chi_sq (double x, double df, double lambda,
       double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the noncentral χ^2-distribution with ν degrees of freedom and noncentrality parameter λ, $P(X \leq x : \nu;\lambda)$, is defined by

$$P(X \leq x : \nu;\lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} P(X \leq x : \nu + 2j;0), \tag{1}$$

where $P(X \leq x : \nu + 2j;0)$ is a central χ^2-distribution with $\nu + 2j$ degrees of freedom.

The value of j at which the Poisson weight, $e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}$, is greatest is determined and the summation (1) is made forward and backward from that value of j.

The recursive relationship:

$$P(X \leq x : a + 2;0) = P(X \leq x : a;0) - \frac{(x^a/2)e^{-x/2}}{\Gamma(a+1)} \tag{2}$$

is used during the summation in (1).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the deviate from the noncentral χ^2-distribution with ν degrees of freedom and noncentrality parameter λ.

Constraint: **x** ≥ 0.0.

2: **df** – double *Input*

On entry: ν, the degrees of freedom of the noncentral χ^2-distribution.

Constraint: **df** ≥ 0.0.

3: **lambda** – double *Input*

On entry: λ, the noncentrality parameter of the noncentral χ^2-distribution.

Constraint: **lambda** ≥ 0.0 if **df** > 0.0 or **lambda** > 0.0 if **df** $= 0.0$.

4: **tol** – double *Input*

On entry: the required accuracy of the solution. If nag_prob_non_central_chi_sq (g01gcc) is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see nag_machine_precision (X02AJC)), then the value of $10 \times$ ***machine precision*** is used instead.

5: **max_iter** – Integer *Input*

On entry: the maximum number of iterations to be performed.

Suggested value: 100. See Section 9 for further discussion.

Constraint: **max_iter** ≥ 1.

6: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_2_REAL_ARG_CONS

On entry, **df** $= 0.0$ and **lambda** $= 0.0$.
Constraint: **lambda** > 0.0 if **df** $= 0.0$.

NE_CHI_PROB

The calculations for the central chi-square probability has failed to converge. A larger value of **tol** should be used.

NE_CONV

The solution has failed to converge in $\langle value \rangle$ iterations. Consider increasing **max_iter** or **tol**.

NE_INT_ARG_LT

On entry, **max_iter** $= \langle value \rangle$.
Constraint: **max_iter** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_POISSON_WEIGHT

The initial value of the Poisson weight used in the summation of (1) (see Section 3) was too small to be calculated. The computed probability is likely to be zero.

NE_REAL_ARG_LT

On entry, **df** $= \langle value \rangle$.
Constraint: **df** ≥ 0.0.

On entry, **lambda** $= \langle value \rangle$.
Constraint: **lambda** ≥ 0.0.

On entry, **x** $= \langle value \rangle$.
Constraint: **x** ≥ 0.0.

NE_TERM_LARGE

The value of a term required in (2) (see Section 3) is too large to be evaluated accurately. The most likely cause of this error is both **x** and **lambda** are too large.

7 Accuracy

The summations described in Section 3 are made until an upper bound on the truncation error relative to the current summation value is less than **tol**.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The number of terms in (1) required for a given accuracy will depend on the following factors:

(i) The rate at which the Poisson weights tend to zero. This will be slower for larger values of λ.

(ii) The rate at which the central χ^2 probabilities tend to zero. This will be slower for larger values of ν and x.

10 Example

This example reads values from various noncentral χ^2-distributions, calculates the lower tail probabilities and prints all these values until the end of data is reached.

10.1 Program Text

```
/* nag_prob_non_central_chi_sq (g01gcc) Example Program.
 *
 * Copyright 1999 Numerical Algorithms Group.
 *
 * Mark 6a revised, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{
  Integer  exit_status = 0, max_iter;
  NagError fail;
  double   df, lambda, prob, tol, x;

  INIT_FAIL(fail);

  printf(
          "nag_prob_non_central_chi_sq (g01gcc) Example Program Results\n\n");

  /* Skip heading in data file */
  scanf("%*[^\n]");

  printf("\n     x       df      lambda   prob\n\n\n");
  tol = 5e-6;
  max_iter = 50;
  while ((scanf(" %lf %lf %lf %*[^\n] ", &x, &df, &lambda)) != EOF)
    {
      /* nag_prob_non_central_chi_sq (g01gcc).
       * Computes probabilities for the non-central chi^2
       * distribution
       */
      prob = nag_prob_non_central_chi_sq(x, df, lambda, tol, max_iter, &fail);
```

```
      if (fail.code != NE_NOERROR)
        {
          printf(
                  "Error from nag_prob_non_central_chi_sq (g01gcc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%8.3f %8.3f %8.3f %8.4f\n", x, df, lambda, prob);
    }
 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_prob_non_central_chi_sq (g01gcc) Example Program Data
  8.26   20.0   3.5            :x df lambda
  6.2     7.5   2.0            :x df lambda
 55.76   45.0   1.0            :x df lambda
```

10.3 Program Results

```
nag_prob_non_central_chi_sq (g01gcc) Example Program Results


     x       df      lambda   prob


   8.260   20.000    3.500   0.0032
   6.200    7.500    2.000   0.2699
  55.760   45.000    1.000   0.8443
```