NAG Library Function Document

nag_prob_non_central_students_t (g01gbc)

1 Purpose

nag_prob_non_central_students_t (g01gbc) returns the lower tail probability for the noncentral Student's t-distribution.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_prob_non_central_students_t (double t, double df, double delta,
       double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the noncentral Student's t-distribution with ν degrees of freedom and noncentrality parameter δ, $P(T \leq t : \nu;\delta)$, is defined by

$$P(T \leq t : \nu;\delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2}\, dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma\left(\frac{1}{2}\nu\right) 2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}.$$

The probability is computed in one of two ways.

(i) When $t = 0.0$, the relationship to the normal is used:

$$P(T \leq t : \nu;\delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2}\, du.$$

(ii) Otherwise the series expansion described in Equation 9 of Amos (1964) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 References

Amos D E (1964) Representations of the central and non-central t-distributions *Biometrika* **51** 451–458

5 Arguments

1: **t** – double *Input*

On entry: t, the deviate from the Student's t-distribution with ν degrees of freedom.

2: **df** – double *Input*

On entry: ν, the degrees of freedom of the Student's t-distribution.

Constraint: **df** ≥ 1.0.

3: **delta** – double *Input*

On entry: δ, the noncentrality argument of the Students t-distribution.

4: **tol** – double *Input*

On entry: the absolute accuracy required by you in the results. If nag_prob_non_central_students_t (g01gbc) is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see nag_machine_precision (X02AJC)), then the value of $10 \times$ ***machine precision*** is used instead.

5: **max_iter** – Integer *Input*

On entry: the maximum number of terms that are used in each of the summations.

Suggested value: 100. See Section 9 for further comments.

Constraint: **max_iter** ≥ 1.

6: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_INT_ARG_LT

On entry, **max_iter** $= \langle value \rangle$.
Constraint: **max_iter** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_PROB_LIMIT

The probability is too close to 0 or 1.

NE_PROBABILITY

The probability is too small to calculate accurately.

NE_REAL_ARG_LT

On entry, **df** $= \langle value \rangle$.
Constraint: **df** ≥ 1.0.

NE_SERIES

One of the series has failed to converge with **max_iter** $= \langle value \rangle$ and **tol** $= \langle value \rangle$. Reconsider the requested tolerance and/or the maximum number of iterations.

7 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than **tol**. There may also be some loss of accuracy due to calculation of gamma functions.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The rate of convergence of the series depends, in part, on the quantity $t^2/(t^2+\nu)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the t-distribution to the F-distribution can be used:

$$F = T^2, \lambda = \delta^2, \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to nag_prob_non_central_f_dist (g01gdc).

Note that nag_prob_non_central_students_t (g01gbc) only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

10 Example

This example reads values from, and degrees of freedom for, and noncentrality arguments of the noncentral Student's t-distributions, calculates the lower tail probabilities and prints all these values until the end of data is reached.

10.1 Program Text

```
/* nag_prob_non_central_students_t (g01gbc) Example Program.
 *
 * Copyright 1999 Numerical Algorithms Group.
 *
 * Mark 6a revised, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{
  Integer  exit_status = 0, max_iter;
  NagError fail;
  double   delta, df, prob, t, tol;

  INIT_FAIL(fail);

  printf(
          "nag_prob_non_central_students_t (g01gbc) Example Program Results"
          "\n\n");

  /* Skip heading in data file */
  scanf("%*[^\n]");

  printf("      t       df    delta    prob\n\n");
  tol = 5e-6;
  max_iter = 50;
  while ((scanf("%lf %lf %lf %*[^\n]", &t, &df, &delta)) != EOF)
    {
      /* nag_prob_non_central_students_t (g01gbc).
       * Computes probabilities for the non-central Student's
       * t-distribution
       */
      prob = nag_prob_non_central_students_t(t, df, delta, tol, max_iter,
                                             &fail);
      if (fail.code == NE_NOERROR)
        printf(" %8.3f%8.3f%8.3f%8.4f\n", t, df, delta, prob);
      else
        {
          printf(
                  "Error from nag_prob_non_central_students_t (g01gbc).\n%s\n",
                  fail.message);
```

```
          exit_status = 1;
          goto END;
        }
    }
 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_prob_non_central_students_t (g01gbc) Example Program Data
 -1.528  20.0  2.0           :t df delta
 -0.188   7.5  1.0           :t df delta
  1.138  45.0  0.0           :t df delta
```

10.3 Program Results

```
nag_prob_non_central_students_t (g01gbc) Example Program Results

      t       df    delta    prob

   -1.528  20.000   2.000  0.0003
   -0.188   7.500   1.000  0.1189
    1.138  45.000   0.000  0.8694
```