

## NAG Library Function Document

### nag\_deviates\_chi\_sq (g01fcc)

#### 1 Purpose

nag\_deviates\_chi\_sq (g01fcc) returns the deviate associated with the given lower tail probability of the  $\chi^2$ -distribution with real degrees of freedom.

#### 2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_chi_sq (double p, double df, NagError *fail)
```

#### 3 Description

The deviate,  $x_p$ , associated with the lower tail probability  $p$  of the  $\chi^2$ -distribution with  $\nu$  degrees of freedom is defined as the solution to

$$P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{\nu/2-1} dX, \quad 0 \leq x_p < \infty; \nu > 0.$$

The required  $x_p$  is found by using the relationship between a  $\chi^2$ -distribution and a gamma distribution, i.e., a  $\chi^2$ -distribution with  $\nu$  degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter  $\nu/2$ .

For very large values of  $\nu$ , greater than  $10^5$ , Wilson and Hilferty's normal approximation to the  $\chi^2$  is used; see Kendall and Stuart (1969).

#### 4 References

Best D J and Roberts D E (1975) Algorithm AS 91. The percentage points of the  $\chi^2$  distribution *Appl. Statist.* **24** 385–388

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

#### 5 Arguments

- |    |   |                     |
|----|---|---------------------|
| 1: | <b>p</b> – double   | <i>Input</i>        |
|    | <i>On entry:</i> $p$ , the lower tail probability from the required $\chi^2$ -distribution. |                     |
|    | <i>Constraint:</i> $0.0 \leq \mathbf{p} < 1.0$ .  |                     |
| 2: | <b>df</b> – double  | <i>Input</i>        |
|    | <i>On entry:</i> $\nu$ , the degrees of freedom of the $\chi^2$ -distribution.              |                     |
|    | <i>Constraint:</i> <b>df</b> > 0.0.   |                     |
| 3: | <b>fail</b> – NagError *  | <i>Input/Output</i> |
|    | The NAG error argument (see Section 3.6 in the Essential Introduction).                     |                     |

## 6 Error Indicators and Warnings

On any of the error conditions listed below except **fail.code** = NE\_ALG\_NOT\_CONV nag\_deviates\_chi\_sq (g01fcc) returns 0.0.

### NE\_ALG\_NOT\_CONV

The algorithm has failed to converge in  $\langle value \rangle$  iterations. The result should be a reasonable approximation.

### NE\_GAM\_NOT\_CONV

The series used to calculate the gamma function has failed to converge. This is an unlikely error exit.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

### NE\_PROBAB\_CLOSE\_TO\_TAIL

The probability is too close to 0.0 or 1.0.

### NE\_REAL\_ARG\_GE

On entry,  $p = \langle value \rangle$ .  
Constraint:  $p < 1.0$ .

### NE\_REAL\_ARG\_LE

On entry,  $df = \langle value \rangle$ .  
Constraint:  $df > 0.0$ .

### NE\_REAL\_ARG\_LT

On entry,  $p = \langle value \rangle$ .  
Constraint:  $p \geq 0.0$ .

## 7 Accuracy

The results should be accurate to five significant digits for most argument values. Some accuracy is lost for  $p$  close to 0.0.

## 8 Parallelism and Performance

Not applicable.

## 9 Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to nag\_deviates\_gamma\_dist (g01ffc) made.

## 10 Example

This example reads lower tail probabilities for several  $\chi^2$ -distributions, and calculates and prints the corresponding deviates until the end of data is reached.

## 10.1 Program Text

```

/* nag_deviates_chi_sq (g01fcc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer    exit_status = 0;
    double     df, p, x;
    NagError   fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    scanf("%*[^\\n]");
    printf("nag_deviates_chi_sq (g01fcc) Example Program Results\\n");
    printf("      p          df          x\\n\\n");
    while (scanf("%lf %lf", &p, &df) != EOF)
    {
        /* nag_deviates_chi_sq (g01fcc).
         * Deviates for the chi^2 distribution
         */
        x = nag_deviates_chi_sq(p, df, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_deviates_chi_sq (g01fcc).\\n%s\\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%8.3f%8.3f%8.3f\\n", p, df, x);
    }

    END:
    return exit_status;
}

```

## 10.2 Program Data

```

nag_deviates_chi_sq (g01fcc) Example Program Data
0.0100 20.0
0.4279 7.50
0.8694 45.0

```

## 10.3 Program Results

```

nag_deviates_chi_sq (g01fcc) Example Program Results
      p          df          x

    0.010  20.000   8.260
    0.428   7.500   6.200
    0.869  45.000  55.759

```

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