NAG Library Function Document

nag_deviates_students_t (g01fbc)

1 Purpose

nag_deviates_students_t (g01fbc) returns the deviate associated with the given tail probability of Student's t-distribution with real degrees of freedom.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_students_t (Nag_TailProbability tail, double p,
       double df, NagError *fail)
```

3 Description

The deviate, t_p associated with the lower tail probability, p, of the Student's t-distribution with ν degrees of freedom is defined as the solution to

$$P\big(T < t_p : \nu\big) = p = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT, \quad \nu \geq 1; \; -\infty < t_p < \infty.$$

For $\nu = 1$ or 2 the integral equation is easily solved for t_p.

For other values of $\nu < 3$ a transformation to the beta distribution is used and the result obtained from nag_deviates_beta (g01fec).

For $\nu \geq 3$ an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's t-distribution *Comm. ACM* **13(10)** 617–619

5 Arguments

1: **tail** – Nag_TailProbability *Input*

On entry: indicates which tail the supplied probability represents.

tail = Nag_UpperTail
The upper tail probability, i.e., $P\big(T \geq t_p : \nu\big)$.

tail = Nag_LowerTail
The lower tail probability, i.e., $P\big(T \leq t_p : \nu\big)$.

tail = Nag_TwoTailSignif
The two tail (significance level) probability, i.e., $P\big(T \geq \big|t_p\big| : \nu\big) + P\big(T \leq -\big|t_p\big| : \nu\big)$.

tail = Nag_TwoTailConfid
The two tail (confidence interval) probability, i.e., $P\big(T \leq \big|t_p\big| : \nu\big) - P\big(T \leq -\big|t_p\big| : \nu\big)$.

Constraint: **tail** = Nag_UpperTail, Nag_LowerTail, Nag_TwoTailSignif or Nag_TwoTailConfid.

2: **p** – double *Input*

On entry: p, the probability from the required Student's t-distribution as defined by **tail**.

Constraint: $0.0 < \mathbf{p} < 1.0$.

3: **df** – double *Input*

On entry: ν, the degrees of freedom of the Student's t-distribution.

Constraint: $\mathbf{df} \geq 1.0$.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

On any of the error conditions listed below except **fail.code** = NE_SOL_NOT_CONV nag_deviates_students_t (g01fbc) returns 0.0.

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GE

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{p} < 1.0$.

NE_REAL_ARG_LE

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{p} > 0.0$.

NE_REAL_ARG_LT

On entry, $\mathbf{df} = \langle value \rangle$.
Constraint: $\mathbf{df} \geq 1.0$.

NE_SOL_NOT_CONV

The solution has failed to converge. However, the result should be a reasonable approximation.

7 Accuracy

The results should be accurate to five significant digits, for most argument values. The error behaviour for various argument values is discussed in Hill (1970).

8 Parallelism and Performance

Not applicable.

9 Further Comments

The value t_p may be calculated by using the transformation described in Section 3 and using nag_deviates_beta (g01fec). This function allows you to set the required accuracy.

10 Example

This example reads the probability, the tail that probability represents and the degrees of freedom for a number of Student's t-distributions and computes the corresponding deviates.

10.1 Program Text

```
/* nag_deviates_students_t (g01fbc) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 * Mark 5 revised, 1998.
 * Mark 7 revised, 2001.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
  Integer                   exit_status=0;
  double                    df, p, t;
  int                       i;
  static Nag_TailProbability tail[] = { Nag_LowerTail, Nag_UpperTail,
                                        Nag_TwoTailSignif, Nag_TwoTailConfid };
  static const char         *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
                                            "Nag_TwoTailSignif",
                                            "Nag_TwoTailConfid" };
  NagError                  fail;

  INIT_FAIL(fail);

  printf(
          "nag_deviates_students_t (g01fbc) Example Program Results\n\n");
  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("    p      df          tail                  t\n\n");
  while (scanf("%lf %lf %d", &p, &df, &i) != EOF)
    {
      /* nag_deviates_students_t (g01fbc).
       * Deviates for Student's t-distribution
       */
      t = nag_deviates_students_t(tail[i], p, df, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_deviates_students_t (g01fbc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%8.3f%8.3f   %-19s  %8.3f\n", p, df, tailmess[i], t);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_deviates_students_t (g01fbc) Example Program Data
0.0100  20.0  2
0.01     7.5  0
0.99    45.0  3
```

10.3 Program Results

```
nag_deviates_students_t (g01fbc) Example Program Results

    p      df          tail                  t

   0.010  20.000   Nag_TwoTailSignif       2.845
   0.010   7.500   Nag_LowerTail          -2.943
   0.990  45.000   Nag_TwoTailConfid       2.690
```