NAG Library Function Document

nag_superlu_solve_lu (f11mfc)

1 Purpose

nag_superlu_solve_lu (f11mfc) solves a real sparse system of linear equations with multiple right-hand sides given an LU factorization of the sparse matrix computed by nag_superlu_lu_factorize (f11mec).

2 Specification

```
#include <nag.h>
#include <nagf11.h>
```

```
void nag_superlu_solve_lu (Nag_OrderType order, Nag_TransType trans,
     Integer n, const Integer iprm[], const Integer il[],
     const double lval[], const Integer iu[], const double uval[],
     Integer nrhs, double b[], Integer pdb, NagError *fail)
```

3 Description

nag_superlu_solve_lu (f11mfc) solves a real system of linear equations with multiple right-hand sides $AX = B$ or $A^{\mathrm{T}}X = B$, according to the value of the argument **trans**, where the matrix factorization $P_rAP_c = LU$ corresponds to an LU decomposition of a sparse matrix stored in compressed column (Harwell–Boeing) format, as computed by nag_superlu_lu_factorize (f11mec).

In the above decomposition L is a lower triangular sparse matrix with unit diagonal elements and U is an upper triangular sparse matrix; P_r and P_c are permutation matrices.

4 References

None.

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **trans** – Nag_TransType *Input*

On entry: specifies whether $AX = B$ or $A^{\mathrm{T}}X = B$ is solved.

trans = Nag_NoTrans
$AX = B$ is solved.

trans = Nag_Trans
$A^{\mathrm{T}}X = B$ is solved.

Constraint: **trans** = Nag_NoTrans or Nag_Trans.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: **iprm**[**7** × **n**] – const Integer *Input*

On entry: the column permutation which defines P_c, the row permutation which defines P_r, plus associated data structures as computed by nag_superlu_lu_factorize (f11mec).

5: **il**[dim] – const Integer *Input*

Note: the dimension, *dim*, of the array **il** must be at least as large as the dimension of the array of the same name in nag_superlu_lu_factorize (f11mec).

On entry: records the sparsity pattern of matrix L as computed by nag_superlu_lu_factorize (f11mec).

6: **lval**[dim] – const double *Input*

Note: the dimension, *dim*, of the array **lval** must be at least as large as the dimension of the array of the same name in nag_superlu_lu_factorize (f11mec).

On entry: records the nonzero values of matrix L and some nonzero values of matrix U as computed by nag_superlu_lu_factorize (f11mec).

7: **iu**[dim] – const Integer *Input*

Note: the dimension, *dim*, of the array **iu** must be at least as large as the dimension of the array of the same name in nag_superlu_lu_factorize (f11mec).

On entry: records the sparsity pattern of matrix U as computed by nag_superlu_lu_factorize (f11mec).

8: **uval**[dim] – const double *Input*

Note: the dimension, *dim*, of the array **uval** must be at least as large as the dimension of the array of the same name in nag_superlu_lu_factorize (f11mec).

On entry: records some nonzero values of matrix U as computed by nag_superlu_lu_factorize (f11mec).

9: **nrhs** – Integer *Input*

On entry: $nrhs$, the number of right-hand sides in B.

Constraint: **nrhs** ≥ 0.

10: **b**[dim] – double *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least

$\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{n} \times \mathbf{pdb})$ when **order** = Nag_RowMajor.

The (i, j)th element of the matrix B is stored in

$\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the **n** by **nrhs** right-hand side matrix B.

On exit: the **n** by **nrhs** solution matrix X.

11: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = Nag_ColMajor, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = Nag_RowMajor, **pdb** $\geq \max(1, \mathbf{nrhs})$.

12: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pdb** = ⟨*value*⟩.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pdb** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩ and **nrhs** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_INVALID_PERM_COL

Incorrect column permutations in array **iprm**.

NE_INVALID_PERM_ROW

Incorrect Row Permutations in array **iprm**.

7 Accuracy

For each right-hand side vector *b*, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***, when partial pivoting is used.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}(A^{\mathrm{T}})$ can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_superlu_refine_lu (f11mhc), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_superlu_condition_number_lu (f11mgc).

8 Parallelism and Performance

nag_superlu_solve_lu (f11mfc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_superlu_solve_lu (f11mfc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

nag_superlu_solve_lu (f11mfc) may be followed by a call to nag_superlu_refine_lu (f11mhc) to refine the solution and return an error estimate.

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 2.00 & 1.00 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & -1.00 & 0 \\ 4.00 & 0 & 1.00 & 0 & 1.00 \\ 0 & 0 & 0 & 1.00 & 2.00 \\ 0 & -2.00 & 0 & 0 & 3.00 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1.56 & 3.12 \\ -0.25 & -0.50 \\ 3.60 & 7.20 \\ 1.33 & 2.66 \\ 0.52 & 1.04 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_superlu_lu_factorize (f11mec).

10.1 Program Text

```
/* nag_superlu_solve_lu (f11mfc) Example Program.
 *
 * Copyright 2005 Numerical Algorithms Group.
 *
 * Mark 8, 2005.
 */

#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf11.h>


int main(void)
{
  double                     flop, thresh;
  Integer                    exit_status = 0, i, j;
  Integer                    n, nnz, nnzl, nnzu, nrhs, nzlmx, nzlumx, nzumx;
  double                     *a = 0, *lval = 0, *uval = 0, *x = 0;
  Integer                    *icolzp = 0, *il = 0, *iprm = 0, *irowix = 0;
  Integer                    *iu = 0;
  /* Nag types */
  Nag_OrderType              order = Nag_ColMajor;
  Nag_MatrixType             matrix = Nag_GeneralMatrix;
  Nag_DiagType               diag = Nag_NonUnitDiag;
  Nag_ColumnPermutationType ispec;
  Nag_TransType              trans;
  NagError                   fail;

  INIT_FAIL(fail);

  printf("nag_superlu_solve_lu (f11mfc) Example Program Results\n\n");
  /* Skip heading in data file */
  scanf("%*[^\n] ");
```

```
/* Read order of matrix and number of right hand sides */
scanf("%ld%ld%*[^\n] ", &n, &nrhs);
/* Read the matrix A */
if (!(icolzp = NAG_ALLOC(n+1, Integer)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }
for (i = 1; i <= n + 1; ++i)
  {
    scanf("%ld%*[^\n] ", &icolzp[i - 1]);
  }
nnz = icolzp[n] - 1;
/* Allocate memory */
if (!(irowix = NAG_ALLOC(nnz, Integer)) ||
    !(a = NAG_ALLOC(nnz, double)) ||
    !(il = NAG_ALLOC(7*n+8*nnz+4, Integer)) ||
    !(iu = NAG_ALLOC(2*n+8*nnz+1, Integer)) ||
    !(uval = NAG_ALLOC(8*nnz, double)) ||
    !(lval = NAG_ALLOC(8*nnz, double)) ||
    !(iprm = NAG_ALLOC(7*n, Integer)) ||
    !(x = NAG_ALLOC(n*nrhs, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }
for (i = 1; i <= nnz; ++i)
  scanf("%lf%ld%*[^\n] ", &a[i - 1], &irowix[i - 1]);
/* Read the right hand sides */
for (j = 1; j <= nrhs; ++j)
  {
    for (i = 1; i <= n; ++i)
      scanf("%lf", &x[j*n + i - n - 1]);
    scanf("%*[^\n] ");
  }
/* Calculate COLAMD permutation */
ispec = Nag_Sparse_Colamd;
/* nag_superlu_column_permutation (f11mdc).
 * Real sparse nonsymmetric linear systems, setup for
 * nag_superlu_lu_factorize (f11mec)
 */
nag_superlu_column_permutation(ispec, n, icolzp, irowix, iprm, &fail);
if (fail.code != NE_NOERROR)
  {
    printf(
            "Error from nag_superlu_column_permutation (f11mdc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
  }


/* Factorise */
thresh = 1.;
nzlmx = 8*nnz;
nzlumx = 8*nnz;
nzumx = 8*nnz;
/* nag_superlu_lu_factorize (f11mec).
 * LU factorization of real sparse matrix
 */
nag_superlu_lu_factorize(n, irowix, a, iprm, thresh, nzlmx, &nzlumx, nzumx,
                         il, lval, iu, uval, &nnzl, &nnzu, &flop, &fail);
if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_superlu_lu_factorize (f11mec).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
  }
```

```
  /* Solve */
  trans = Nag_NoTrans;
  /* nag_superlu_solve_lu (f11mfc).
   * Solution of real sparse simultaneous linear equations
   * (coefficient matrix already factorized)
   */
  nag_superlu_solve_lu(order, trans, n, iprm, il, lval, iu, uval, nrhs, x,
                       n, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_superlu_solve_lu (f11mfc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
  /* Output results */
  printf("\n");
  /* nag_gen_real_mat_print (x04cac).
   * Print real general matrix (easy-to-use)
   */
  fflush(stdout);
  nag_gen_real_mat_print(order, matrix, diag, n, nrhs,
                         x, n, "Solutions", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

 END:
  NAG_FREE(a);
  NAG_FREE(lval);
  NAG_FREE(uval);
  NAG_FREE(x);
  NAG_FREE(icolzp);
  NAG_FREE(il);
  NAG_FREE(iprm);
  NAG_FREE(irowix);
  NAG_FREE(iu);

  return exit_status;
}
```

10.2 Program Data

```
nag_superlu_solve_lu (f11mfc) Example Program Data
  5 2  n, nrhs
 1
 3
 5
 7
 9
 12   icolzp(i) i=0..n
  2.   1
  4.   3
  1.   1
 -2.   5
  1.   2
  1.   3
 -1.   2
  1.   4
  1.   3
  2.   4
  3.   5    a(i), irowix(i) i=0..nnz-1
 1.56 -.25 3.6 1.33 .52
 3.12 -.50 7.2 2.66 1.04 matrix x
```

10.3 Program Results

```
nag_superlu_solve_lu (f11mfc) Example Program Results


 Solutions
             1          2
 1      0.7000     1.4000
 2      0.1600     0.3200
 3      0.5200     1.0400
 4      0.7700     1.5400
 5      0.2800     0.5600
```