

## NAG Library Function Document

### nag\_dggsvp (f08vec)

#### 1 Purpose

nag\_dggsvp (f08vec) uses orthogonal transformations to simultaneously reduce the  $m$  by  $n$  matrix  $A$  and the  $p$  by  $n$  matrix  $B$  to upper triangular form. This factorization is usually used as a preprocessing step for computing the generalized singular value decomposition (GSVD).

#### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_dggsvp (Nag_OrderType order, Nag_ComputeUType jobu,
  Nag_ComputeVType jobv, Nag_ComputeQType jobq, Integer m, Integer p,
  Integer n, double a[], Integer pda, double b[], Integer pdb,
  double tola, double tolb, Integer *k, Integer *l, double u[],
  Integer pdu, double v[], Integer pdv, double q[], Integer pdq,
  NagError *fail)
```

#### 3 Description

nag\_dggsvp (f08vec) computes orthogonal matrices  $U$ ,  $V$  and  $Q$  such that

$$U^T A Q = \begin{cases} \begin{matrix} & n-k-l & k & l \\ & k \begin{pmatrix} 0 & A_{12} & A_{13} \\ & 0 & 0 \\ & 0 & 0 \end{pmatrix} \\ & l \begin{pmatrix} 0 & 0 & A_{23} \\ & 0 & 0 \end{pmatrix} \\ m-k-l \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \end{matrix}, & \text{if } m-k-l \geq 0; \\ \begin{matrix} & n-k-l & k & l \\ & k \begin{pmatrix} 0 & A_{12} & A_{13} \\ & 0 & 0 \end{pmatrix} \\ m-k \begin{pmatrix} 0 & 0 & A_{23} \end{pmatrix} \end{matrix}, & \text{if } m-k-l < 0; \end{cases}$$

$$V^T B Q = \begin{matrix} & n-k-l & k & l \\ l \begin{pmatrix} 0 & 0 & B_{13} \\ & 0 & 0 \end{pmatrix} \\ p-l \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

where the  $k$  by  $k$  matrix  $A_{12}$  and  $l$  by  $l$  matrix  $B_{13}$  are nonsingular upper triangular;  $A_{23}$  is  $l$  by  $l$  upper triangular if  $m-k-l \geq 0$  and is  $(m-k)$  by  $l$  upper trapezoidal otherwise.  $(k+l)$  is the effective numerical rank of the  $(m+p)$  by  $n$  matrix  $(A^T \ B^T)^T$ .

This decomposition is usually used as the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see function nag\_dggsvd (f08vac).

## 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
  
- 2: **jobu** – Nag\_ComputeUType *Input*  
*On entry:* if **jobu** = Nag\_AllU, the orthogonal matrix  $U$  is computed.  
 If **jobu** = Nag\_NotU,  $U$  is not computed.  
*Constraint:* **jobu** = Nag\_AllU or Nag\_NotU.
  
- 3: **jobv** – Nag\_ComputeVType *Input*  
*On entry:* if **jobv** = Nag\_ComputeV, the orthogonal matrix  $V$  is computed.  
 If **jobv** = Nag\_NotV,  $V$  is not computed.  
*Constraint:* **jobv** = Nag\_ComputeV or Nag\_NotV.
  
- 4: **jobq** – Nag\_ComputeQType *Input*  
*On entry:* if **jobq** = Nag\_ComputeQ, the orthogonal matrix  $Q$  is computed.  
 If **jobq** = Nag\_NotQ,  $Q$  is not computed.  
*Constraint:* **jobq** = Nag\_ComputeQ or Nag\_NotQ.
  
- 5: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $A$ .  
*Constraint:*  $m \geq 0$ .
  
- 6: **p** – Integer *Input*  
*On entry:*  $p$ , the number of rows of the matrix  $B$ .  
*Constraint:*  $p \geq 0$ .
  
- 7: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrices  $A$  and  $B$ .  
*Constraint:*  $n \geq 0$ .
  
- 8: **a**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  
 $\max(1, \mathbf{pda} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pda})$  when **order** = Nag\_RowMajor.

The  $(i, j)$ th element of the matrix  $A$  is stored in

$$\begin{aligned} & \mathbf{a}[(j-1) \times \mathbf{pda} + i - 1] \text{ when } \mathbf{order} = \text{Nag\_ColMajor}; \\ & \mathbf{a}[(i-1) \times \mathbf{pda} + j - 1] \text{ when } \mathbf{order} = \text{Nag\_RowMajor}. \end{aligned}$$

*On entry:* the  $m$  by  $n$  matrix  $A$ .

*On exit:* contains the triangular (or trapezoidal) matrix described in Section 3.

9: **pda** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **a**.

*Constraints:*

$$\begin{aligned} & \text{if } \mathbf{order} = \text{Nag\_ColMajor}, \mathbf{pda} \geq \max(1, \mathbf{m}); \\ & \text{if } \mathbf{order} = \text{Nag\_RowMajor}, \mathbf{pda} \geq \max(1, \mathbf{n}). \end{aligned}$$

10: **b**[*dim*] – double *Input/Output*

**Note:** the dimension, *dim*, of the array **b** must be at least

$$\begin{aligned} & \max(1, \mathbf{pdb} \times \mathbf{n}) \text{ when } \mathbf{order} = \text{Nag\_ColMajor}; \\ & \max(1, \mathbf{p} \times \mathbf{pdb}) \text{ when } \mathbf{order} = \text{Nag\_RowMajor}. \end{aligned}$$

The  $(i, j)$ th element of the matrix  $B$  is stored in

$$\begin{aligned} & \mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1] \text{ when } \mathbf{order} = \text{Nag\_ColMajor}; \\ & \mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1] \text{ when } \mathbf{order} = \text{Nag\_RowMajor}. \end{aligned}$$

*On entry:* the  $p$  by  $n$  matrix  $B$ .

*On exit:* contains the triangular matrix described in Section 3.

11: **pdb** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.

*Constraints:*

$$\begin{aligned} & \text{if } \mathbf{order} = \text{Nag\_ColMajor}, \mathbf{pdb} \geq \max(1, \mathbf{p}); \\ & \text{if } \mathbf{order} = \text{Nag\_RowMajor}, \mathbf{pdb} \geq \max(1, \mathbf{n}). \end{aligned}$$

12: **tola** – double *Input*

13: **tolb** – double *Input*

*On entry:* **tola** and **tolb** are the thresholds to determine the effective numerical rank of matrix  $B$  and a subblock of  $A$ . Generally, they are set to

$$\begin{aligned} \mathbf{tola} &= \max(\mathbf{m}, \mathbf{n}) \|A\| \epsilon, \\ \mathbf{tolb} &= \max(\mathbf{p}, \mathbf{n}) \|B\| \epsilon, \end{aligned}$$

where  $\epsilon$  is the *machine precision*.

The size of **tola** and **tolb** may affect the size of backward errors of the decomposition.

14: **k** – Integer \* *Output*

15: **l** – Integer \* *Output*

*On exit:* **k** and **l** specify the dimension of the subblocks  $k$  and  $l$  as described in Section 3;  $(k + l)$  is the effective numerical rank of  $(\mathbf{a}^T \ \mathbf{b}^T)^T$ .

- 16: **u**[*dim*] – double *Output*
- Note:** the dimension, *dim*, of the array **u** must be at least  
 $\max(1, \mathbf{pdu} \times \mathbf{m})$  when **jobu** = Nag\_AllU;  
 1 otherwise.
- The (*i*, *j*)th element of the matrix *U* is stored in  
 $\mathbf{u}[(j-1) \times \mathbf{pdu} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{u}[(i-1) \times \mathbf{pdu} + j - 1]$  when **order** = Nag\_RowMajor.
- On exit:* if **jobu** = Nag\_AllU, **u** contains the orthogonal matrix *U*.  
 If **jobu** = Nag\_NotU, **u** is not referenced.
- 17: **pdu** – Integer *Input*
- On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **u**.
- Constraints:*  
 if **jobu** = Nag\_AllU, **pdu**  $\geq \max(1, \mathbf{m})$ ;  
 otherwise **pdu**  $\geq 1$ .
- 18: **v**[*dim*] – double *Output*
- Note:** the dimension, *dim*, of the array **v** must be at least  
 $\max(1, \mathbf{pdv} \times \mathbf{p})$  when **jobv** = Nag\_ComputeV;  
 1 otherwise.
- The (*i*, *j*)th element of the matrix *V* is stored in  
 $\mathbf{v}[(j-1) \times \mathbf{pdv} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{v}[(i-1) \times \mathbf{pdv} + j - 1]$  when **order** = Nag\_RowMajor.
- On exit:* if **jobv** = Nag\_ComputeV, **v** contains the orthogonal matrix *V*.  
 If **jobv** = Nag\_NotV, **v** is not referenced.
- 19: **pdv** – Integer *Input*
- On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **v**.
- Constraints:*  
 if **jobv** = Nag\_ComputeV, **pdv**  $\geq \max(1, \mathbf{p})$ ;  
 otherwise **pdv**  $\geq 1$ .
- 20: **q**[*dim*] – double *Output*
- Note:** the dimension, *dim*, of the array **q** must be at least  
 $\max(1, \mathbf{pdq} \times \mathbf{n})$  when **jobq** = Nag\_ComputeQ;  
 1 otherwise.
- The (*i*, *j*)th element of the matrix *Q* is stored in  
 $\mathbf{q}[(j-1) \times \mathbf{pdq} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{q}[(i-1) \times \mathbf{pdq} + j - 1]$  when **order** = Nag\_RowMajor.
- On exit:* if **jobq** = Nag\_ComputeQ, **q** contains the orthogonal matrix *Q*.  
 If **jobq** = Nag\_NotQ, **q** is not referenced.

- 21: **pdq** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **q**.  
*Constraints:*  
 if **jobq** = Nag\_ComputeQ, **pdq**  $\geq$  max(1, **n**);  
 otherwise **pdq**  $\geq$  1.
- 22: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_ENUM\_INT\_2

On entry, **jobq** =  $\langle value \rangle$ , **pdu** =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .  
 Constraint: if **jobq** = Nag\_ComputeQ, **pdu**  $\geq$  max(1, **n**);  
 otherwise **pdu**  $\geq$  1.

On entry, **jobu** =  $\langle value \rangle$ , **pdu** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .  
 Constraint: if **jobu** = Nag\_AllU, **pdu**  $\geq$  max(1, **m**);  
 otherwise **pdu**  $\geq$  1.

On entry, **jobv** =  $\langle value \rangle$ , **pdu** =  $\langle value \rangle$  and **p** =  $\langle value \rangle$ .  
 Constraint: if **jobv** = Nag\_ComputeV, **pdu**  $\geq$  max(1, **p**);  
 otherwise **pdu**  $\geq$  1.

On entry, **pdu** =  $\langle value \rangle$ , **jobq** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: if **jobq** = Nag\_ComputeQ, **pdu**  $\geq$  max(1, **n**);  
 otherwise **pdu**  $\geq$  1.

### NE\_INT

On entry, **m** =  $\langle value \rangle$ .  
 Constraint: **m**  $\geq$  0.

On entry, **n** =  $\langle value \rangle$ .  
 Constraint: **n**  $\geq$  0.

On entry, **p** =  $\langle value \rangle$ .  
 Constraint: **p**  $\geq$  0.

On entry, **pda** =  $\langle value \rangle$ .  
 Constraint: **pda**  $>$  0.

On entry, **pdb** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $>$  0.

On entry, **pdu** =  $\langle value \rangle$ .  
 Constraint: **pdu**  $>$  0.

On entry, **pdu** =  $\langle value \rangle$ .  
 Constraint: **pdu**  $>$  0.

On entry, **pdu** =  $\langle value \rangle$ .  
 Constraint: **pdu**  $>$  0.

**NE\_INT\_2**

On entry, **pda** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq \max(1, \mathbf{m})$ .

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq \max(1, \mathbf{n})$ .

On entry, **pdb** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $\geq \max(1, \mathbf{n})$ .

On entry, **pdb** =  $\langle value \rangle$  and **p** =  $\langle value \rangle$ .  
 Constraint: **pdb**  $\geq \max(1, \mathbf{p})$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**7 Accuracy**

The computed factorization is nearly the exact factorization for nearby matrices  $(A + E)$  and  $(B + F)$ , where

$$\|E\|_2 = O(\epsilon)\|A\|_2 \quad \text{and} \quad \|F\|_2 = O(\epsilon)\|B\|_2,$$

and  $\epsilon$  is the *machine precision*.

**8 Parallelism and Performance**

nag\_dggsvp (f08vec) is not threaded by NAG in any implementation.

nag\_dggsvp (f08vec) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The complex analogue of this function is nag\_zggsvp (f08vsc).

**10 Example**

This example finds the generalized factorization

$$A = U\Sigma_1 \begin{pmatrix} \mathbf{0} & S \end{pmatrix} Q^T, \quad B = V\Sigma_2 \begin{pmatrix} \mathbf{0} & T \end{pmatrix} Q^T,$$

of the matrix pair  $(A \ B)$ , where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & -3 & 3 \\ 4 & 6 & 5 \end{pmatrix}.$$

**10.1 Program Text**

```
/* nag_dggsvp (f08vec) Example Program.
 *
 * Copyright 2011 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */
```

```

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagf16.h>
#include <nagx02.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    double      eps, norm, tola, tolb;
    Integer     i, irank, j, k, l, m, n, nrows, p, pda, pdb, pdq, pdu, pdv;
    Integer     exit_status = 0;
    /* Arrays */
    double      *a = 0, *b = 0, *q = 0, *u = 0, *v = 0;

    /* Nag Types */
    NagError    fail;
    Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dggsvp (f08vec) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n]");
    scanf("%ld%ld%ld%*[\n]", &m, &n, &p);
    if (n < 0 || m < 0 || p < 0)
    {
        printf("Invalid n, m or p\n");
        exit_status = 1;
        goto END;
    }

#ifdef NAG_COLUMN_MAJOR
    pda = m;
    pdb = p;
    pdv = p;
#else
    pda = n;
    pdb = n;
    pdv = m;
#endif
    pdq = n;
    pdu = m;

    /* Allocate memory */
    if (!(a = NAG_ALLOC(m*n, double)) ||
        !(b = NAG_ALLOC(p*n, double)) ||
        !(q = NAG_ALLOC(n*n, double)) ||
        !(u = NAG_ALLOC(m*m, double)) ||
        !(v = NAG_ALLOC(p*m, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read the m by n matrix A and p by n matrix B from data file */

```

```

for (i = 1; i <= m; ++i)
  for (j = 1; j <= n; ++j) scanf("%lf", &A(i, j));
scanf("%*[\n]");
for (i = 1; i <= p; ++i)
  for (j = 1; j <= n; ++j) scanf("%lf", &B(i, j));
scanf("%*[\n]");

/* get norms of A and B using nag_dge_norm (f16rac). */
nag_dge_norm(order, Nag_OneNorm, m, n, a, pda, &norm, &fail);
nag_dge_norm(order, Nag_OneNorm, p, n, b, pdb, &norm, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_dge_norm (f16rac).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}

/* Get the machine precision, using nag_machine_precision (x02ajc) */
eps = nag_machine_precision;

tola = MAX(m, n) * norm * eps;
tolb = MAX(p, n) * norm * eps;

/* Compute the factorization of (A, B) (A = U*S*(Q**T), B = V*T*(Q**T))
 * using nag_dggsvp (f08vec).
 */
nag_dggsvp(order, Nag_AllU, Nag_ComputeV, Nag_ComputeQ, m, p, n, a, pda, b,
           pdb, tola, tolb, &k, &l, u, pdu, v, pdv, q, pdq, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_dggsvp (f08vec).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}

/* Print details of the generalized SVD */
irank = k + 1;
printf("Numerical rank of (A**T B**T)**T (K+L)\n%5ld\n\n", irank);
nrows = MIN(m, irank);
fflush(stdout);
nag_gen_real_mat_print_comp(order, Nag_UpperMatrix, Nag_NonUnitDiag, nrows,
                           irank, &A(1, n - irank + 1), pda, "%13.4e",
                           "Matrix S", Nag_IntegerLabels, 0,
                           Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) goto FAIL;
printf("\n");
fflush(stdout);
nag_gen_real_mat_print_comp(order, Nag_UpperMatrix, Nag_NonUnitDiag, 1, 1,
                           &B(1, n - 1 + 1), pdb, "%13.4e",
                           "Upper triangular matrix T", Nag_IntegerLabels,
                           0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) goto FAIL;
printf("\n");
fflush(stdout);
nag_gen_real_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, m,
                           m, u, pdu, "%13.4e", "Orthogonal matrix U",
                           Nag_IntegerLabels, 0, Nag_IntegerLabels, 0,
                           80, 0, 0, &fail);
if (fail.code != NE_NOERROR) goto FAIL;
printf("\n");
fflush(stdout);
nag_gen_real_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, p, p,
                           v, pdv, "%13.4e", "Orthogonal matrix V",
                           Nag_IntegerLabels, 0, Nag_IntegerLabels, 0, 80,
                           0, 0, &fail);
if (fail.code != NE_NOERROR) goto FAIL;
printf("\n");
fflush(stdout);
nag_gen_real_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n,
                           q, pdq, "%13.4e", "Orthogonal matrix Q",
                           Nag_IntegerLabels, 0, Nag_IntegerLabels, 0, 80,

```



```

                                0, 0, &fail);
FAIL:
  if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_gen_real_mat_print_comp (x04cbc).\n%s\n",
          fail.message);
    exit_status = 1;
    goto END;
  }

END:
  NAG_FREE(a);
  NAG_FREE(b);
  NAG_FREE(q);
  NAG_FREE(u);
  NAG_FREE(v);

  return exit_status;
}

```

## 10.2 Program Data

nag\_dggsvp (f08vec) Example Program Data

```

  4    3    2    : m, n and p

  1.0  2.0  3.0
  3.0  2.0  1.0
  4.0  5.0  6.0
  7.0  8.0  8.0 : matrix A

 -2.0 -3.0  3.0
  4.0  6.0  5.0 : matrix B

```

## 10.3 Program Results

nag\_dggsvp (f08vec) Example Program Results

Numerical rank of  $(A^{**T} B^{**T})^{**T}$  (K+L)  
3

Matrix S

	1	2	3
1	-2.0569e+00	1.0771e+01	-7.2814e+00
2		7.1947e+00	-7.5262e+00
3			5.8129e-01

Upper triangular matrix T

	1	2
1	8.0623e+00	-3.1305e+00
2		-4.9193e+00

Orthogonal matrix U

	1	2	3	4
1	-1.3484e-01	5.1025e-01	-2.4351e-01	8.1373e-01
2	6.7420e-01	-5.4670e-01	-3.5349e-01	3.4874e-01
3	2.6968e-01	4.8292e-01	-6.9127e-01	-4.6499e-01
4	6.7420e-01	4.5558e-01	5.8129e-01	1.5127e-15

Orthogonal matrix V

	1	2
1	-4.4721e-01	8.9443e-01
2	8.9443e-01	4.4721e-01

Orthogonal matrix Q

	1	2	3
1	-8.3205e-01	5.5470e-01	0.0000e+00
2	5.5470e-01	8.3205e-01	0.0000e+00

3    0.0000e+00    0.0000e+00    -1.0000e+00

---