NAG Library Function Document

nag_dtrsna (f08qlc)

1 Purpose

nag_dtrsna (f08qlc) estimates condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_dtrsna (Nag_OrderType order, Nag_JobType job,
     Nag_HowManyType how_many, const Nag_Boolean select[], Integer n,
     const double t[], Integer pdt, const double vl[], Integer pdvl,
     const double vr[], Integer pdvr, double s[], double sep[], Integer mm,
     Integer *m, NagError *fail)
```

3 Description

nag_dtrsna (f08qlc) estimates condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T in canonical Schur form. These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix $A = ZTZ^{\mathrm{T}}$ (with orthogonal Z), from which T may have been derived.

nag_dtrsna (f08qlc) computes the reciprocal of the condition number of an eigenvalue λ_i as

$$s_i = \frac{|v^{\mathrm{H}}u|}{\|u\|_E\|v\|_E},$$

where u and v are the right and left eigenvectors of T, respectively, corresponding to λ_i. This reciprocal condition number always lies between zero (i.e., ill-conditioned) and one (i.e., well-conditioned).

An approximate error estimate for a computed eigenvalue λ_i is then given by

$$\frac{\epsilon\|T\|}{s_i},$$

where ϵ is the ***machine precision***.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to λ_i, the function first calls nag_dtrexc (f08qfc) to reorder the eigenvalues so that λ_i is in the leading position:

$$T = Q\begin{pmatrix} \lambda_i & c^{\mathrm{T}} \\ 0 & T_{22} \end{pmatrix}Q^{\mathrm{T}}.$$

The reciprocal condition number of the eigenvector is then estimated as sep_i, the smallest singular value of the matrix $(T_{22} - \lambda_i I)$. This number ranges from zero (i.e., ill-conditioned) to very large (i.e., well-conditioned).

An approximate error estimate for a computed right eigenvector u corresponding to λ_i is then given by

$$\frac{\epsilon\|T\|}{sep_i}.$$

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **job** – Nag_JobType *Input*

On entry: indicates whether condition numbers are required for eigenvalues and/or eigenvectors.

job = Nag_EigVals
Condition numbers for eigenvalues only are computed.

job = Nag_EigVecs
Condition numbers for eigenvectors only are computed.

job = Nag_DoBoth
Condition numbers for both eigenvalues and eigenvectors are computed.

Constraint: **job** = Nag_EigVals, Nag_EigVecs or Nag_DoBoth.

3: **how_many** – Nag_HowManyType *Input*

On entry: indicates how many condition numbers are to be computed.

how_many = Nag_ComputeAll
Condition numbers for all eigenpairs are computed.

how_many = Nag_ComputeSelected
Condition numbers for selected eigenpairs (as specified by **select**) are computed.

Constraint: **how_many** = Nag_ComputeAll or Nag_ComputeSelected.

4: **select**[*dim*] – const Nag_Boolean *Input*

Note: the dimension, *dim*, of the array **select** must be at least

n when **how_many** = Nag_ComputeSelected;
otherwise **select** may be **NULL**.

On entry: specifies the eigenpairs for which condition numbers are to be computed if **how_many** = Nag_ComputeSelected. To select condition numbers for the eigenpair corresponding to the real eigenvalue λ_j, **select**$[j-1]$ must be set Nag_TRUE. To select condition numbers corresponding to a complex conjugate pair of eigenvalues λ_j and λ_{j+1}, **select**$[j-1]$ and/or **select**$[j]$ must be set to Nag_TRUE.

If **how_many** = Nag_ComputeAll, **select** is not referenced and may be **NULL**.

5: **n** – Integer *Input*

On entry: n, the order of the matrix T.

Constraint: **n** ≥ 0.

6: **t**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **t** must be at least **pdt** $\times$ **n**.

The (i,j)th element of the matrix T is stored in

t$[(j-1) \times$ **pdt** $+ i - 1]$ when **order** = Nag_ColMajor;
t$[(i-1) \times$ **pdt** $+ j - 1]$ when **order** = Nag_RowMajor.

On entry: the n by n upper quasi-triangular matrix T in canonical Schur form, as returned by nag_dhseqr (f08pec).

7: **pdt** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **t**.

Constraint: **pdt** $\geq \max(1, \mathbf{n})$.

8: **vl**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **vl** must be at least

pdvl $\times$ **mm** when **job** = Nag_EigVals or Nag_DoBoth and **order** = Nag_ColMajor;
n $\times$ **pdvl** when **job** = Nag_EigVals or Nag_DoBoth and **order** = Nag_RowMajor;
otherwise **vl** may be **NULL**.

The (i, j)th element of the matrix is stored in

vl$[(j-1) \times$ **pdvl** $+ i - 1]$ when **order** = Nag_ColMajor;
vl$[(i-1) \times$ **pdvl** $+ j - 1]$ when **order** = Nag_RowMajor.

On entry: if **job** = Nag_EigVals or Nag_DoBoth, **vl** must contain the left eigenvectors of T (or of any matrix QTQ^{T} with Q orthogonal) corresponding to the eigenpairs specified by **how_many** and **select**. The eigenvectors **must** be stored in consecutive rows or columns of **vl**, as returned by nag_dhsein (f08pkc) or nag_dtrevc (f08qkc).

If **job** = Nag_EigVecs, **vl** is not referenced and may be **NULL**.

9: **pdvl** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **vl**.

Constraints:

if **order** = Nag_ColMajor,

if **job** = Nag_EigVals or Nag_DoBoth, **pdvl** $\geq$ **n**;
if **job** = Nag_EigVecs, **vl** may be **NULL**.;

if **order** = Nag_RowMajor,

if **job** = Nag_EigVals or Nag_DoBoth, **pdvl** $\geq$ **mm**;
if **job** = Nag_EigVecs, **vl** may be **NULL**..

10: **vr**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **vr** must be at least

pdvr $\times$ **mm** when **job** = Nag_EigVals or Nag_DoBoth and **order** = Nag_ColMajor;
n $\times$ **pdvr** when **job** = Nag_EigVals or Nag_DoBoth and **order** = Nag_RowMajor;
otherwise **vr** may be **NULL**.

The (i, j)th element of the matrix is stored in

vr$[(j-1) \times$ **pdvr** $+ i - 1]$ when **order** = Nag_ColMajor;
vr$[(i-1) \times$ **pdvr** $+ j - 1]$ when **order** = Nag_RowMajor.

On entry: if **job** = Nag_EigVals or Nag_DoBoth, **vr** must contain the right eigenvectors of T (or of any matrix QTQ^{T} with Q orthogonal) corresponding to the eigenpairs specified by **how_many** and **select**. The eigenvectors **must** be stored in consecutive rows or columns of **vr**, as returned by nag_dhsein (f08pkc) or nag_dtrevc (f08qkc).

If **job** = Nag_EigVecs, **vr** is not referenced and may be **NULL**.

11: **pdvr** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **vr**.

Constraints:

if **order** = Nag_ColMajor,

if **job** = Nag_EigVals or Nag_DoBoth, **pdvr** $\geq$ **n**;
if **job** = Nag_EigVecs, **vr** may be **NULL**.;

if **order** = Nag_RowMajor,

if **job** = Nag_EigVals or Nag_DoBoth, **pdvr** $\geq$ **mm**;
if **job** = Nag_EigVecs, **vr** may be **NULL**..

12: **s**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **s** must be at least

mm when **job** = Nag_EigVals or Nag_DoBoth;
otherwise **s** may be **NULL**.

On exit: the reciprocal condition numbers of the selected eigenvalues if **job** = Nag_EigVals or Nag_DoBoth, stored in consecutive elements of the array. Thus **s**$[j-1]$, **sep**$[j-1]$ and the jth rows or columns of **vl** and **vr** all correspond to the same eigenpair (but not in general the jth eigenpair unless all eigenpairs have been selected). For a complex conjugate pair of eigenvalues, two consecutive elements of **s** are set to the same value.

If **job** = Nag_EigVecs, **s** is not referenced and may be **NULL**.

13: **sep**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **sep** must be at least

mm when **job** = Nag_EigVecs or Nag_DoBoth;
otherwise **sep** may be **NULL**.

On exit: the estimated reciprocal condition numbers of the selected right eigenvectors if **job** = Nag_EigVecs or Nag_DoBoth, stored in consecutive elements of the array. For a complex eigenvector, two consecutive elements of **sep** are set to the same value. If the eigenvalues cannot be reordered to compute **sep**$[j]$, then **sep**$[j]$ is set to zero; this can only occur when the true value would be very small anyway.

If **job** = Nag_EigVals, **sep** is not referenced and may be **NULL**.

14: **mm** – Integer *Input*

On entry: the number of elements in the arrays **s** and **sep**, and the number of rows or columns (depending on the value of **order**) in the arrays **vl** and **vr** (if used). The precise number required, m, is n if **how_many** = Nag_ComputeAll; if **how_many** = Nag_ComputeSelected, m is obtained by counting 1 for each selected real eigenvalue, and 2 for each selected complex conjugate pair of eigenvalues (see **select**), in which case $0 \leq m \leq n$.

Constraint: **mm** $\geq$ **m**.

Constraint: if **how_many** = Nag_ComputeAll, **mm** $\geq$ **n**.

15: **m** – Integer * *Output*

On exit: m, the number of elements of **s** and/or **sep** actually used to store the estimated condition numbers. If **how_many** = Nag_ComputeAll, **m** is set to n.

16: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument $\langle value\rangle$ had an illegal value.

NE_ENUM_INT_2

On entry, **job** $= \langle value\rangle$, **pdvl** $= \langle value\rangle$, **mm** $= \langle value\rangle$.
Constraint: if **job** = Nag_EigVals or Nag_DoBoth, **pdvl** $\geq$ **mm**.

On entry, **job** $= \langle value\rangle$, **pdvl** $= \langle value\rangle$ and **n** $= \langle value\rangle$.
Constraint: if **job** = Nag_EigVals or Nag_DoBoth, **pdvl** $\geq$ **n**.

On entry, **job** $= \langle value\rangle$, **pdvr** $= \langle value\rangle$, **mm** $= \langle value\rangle$.
Constraint: if **job** = Nag_EigVals or Nag_DoBoth, **pdvr** $\geq$ **mm**.

On entry, **job** $= \langle value\rangle$, **pdvr** $= \langle value\rangle$ and **n** $= \langle value\rangle$.
Constraint: if **job** = Nag_EigVals or Nag_DoBoth, **pdvr** $\geq$ **n**.

On entry, **mm** $= \langle value\rangle$, **n** $= \langle value\rangle$ and **how_many** $= \langle value\rangle$.
Constraint: if **how_many** = Nag_ComputeAll, **mm** $\geq$ **n**.

NE_INT

On entry, **n** $= \langle value\rangle$.
Constraint: **n** ≥ 0.

On entry, **pdt** $= \langle value\rangle$.
Constraint: **pdt** > 0.

On entry, **pdvl** $= \langle value\rangle$.
Constraint: **pdvl** > 0.

On entry, **pdvr** $= \langle value\rangle$.
Constraint: **pdvr** > 0.

NE_INT_2

On entry, **mm** $= \langle value\rangle$ and **m** $= \langle value\rangle$.
Constraint: **mm** $\geq$ **m**.

On entry, **pdt** $= \langle value\rangle$ and **n** $= \langle value\rangle$.
Constraint: **pdt** $\geq \max(1, \mathbf{n})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

The computed values sep_i may over estimate the true value, but seldom by a factor of more than 3.

8 Parallelism and Performance

nag_dtrsna (f08qlc) is not threaded by NAG in any implementation.

nag_dtrsna (f08qlc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

For a description of canonical Schur form, see the document for nag_dhseqr (f08pec).

The complex analogue of this function is nag_ztrsna (f08qyc).

10 Example

This example computes approximate error estimates for all the eigenvalues and right eigenvectors of the matrix T, where

$$T = \begin{pmatrix} 0.7995 & -0.1144 & 0.0060 & 0.0336 \\ 0.0000 & -0.0994 & 0.2478 & 0.3474 \\ 0.0000 & -0.6483 & -0.0994 & 0.2026 \\ 0.0000 & 0.0000 & 0.0000 & -0.1007 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_dtrsna (f08qlc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
  /* Scalars */
  Integer       i, j, m, n, pdt, pdvl, pdvr;
  Integer       s_len;
  Integer       exit_status = 0;
  double        eps, tnorm;
  NagError      fail;
  Nag_OrderType order;
  /* Arrays */
  double        *s = 0, *sep = 0, *t = 0, *vl = 0, *vr = 0;

#ifdef NAG_COLUMN_MAJOR
#define T(I, J) t[(J-1)*pdt + I - 1]
  order = Nag_ColMajor;
#else
#define T(I, J) t[(I-1)*pdt + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_dtrsna (f08qlc) Example Program Results\n");

  /* Skip heading in data file */
  scanf("%*[^\n] ");
  scanf("%ld%*[^\n] ", &n);
#ifdef NAG_COLUMN_MAJOR
  pdt = n;
  pdvl = n;
  pdvr = n;
#else
```

```
  pdt = n;
  pdvl = n;
  pdvr = n;
#endif
  s_len = n;

  /* Allocate memory */
  if (!(t = NAG_ALLOC(n * n, double)) ||
      !(vl = NAG_ALLOC(n * n, double)) ||
      !(vr = NAG_ALLOC(n * n, double)) ||
      !(s = NAG_ALLOC(s_len, double)) ||
      !(sep = NAG_ALLOC(s_len, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read T from data file */
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= n; ++j)
        scanf("%lf", &T(i, j));
    }
  scanf("%*[^\n] ");

  /* Calculate right and left eigrnvectors of real upper quasi-triangular
   * matrix T using nag_dtrevc (f08qkc).
   */
  nag_dtrevc(order, Nag_BothSides, Nag_ComputeAll, NULL, n, t, pdt,
             vl, pdvl, vr, pdvr, n, &m, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dtrevc (f08qkc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Estimate condition numbers for all the eigenvalues and
   * right eigenvectors of real upper quasi-triangular matrix T using
   * nag_dtrsna (f08qlc).
   */
  nag_dtrsna(order, Nag_DoBoth, Nag_ComputeAll, NULL, n, t, pdt,
             vl, pdvl, vr, pdvr, s, sep, n, &m, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dtrsna (f08qlc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print condition numbers of eigenvalues and right eigenvectors */
  printf("\nS\n");
  for (i = 0; i < n; ++i)
    printf("%11.1e", s[i]);
  printf("\n\nSep\n");
  for (i = 0; i < n; ++i)
    printf("%11.1e", sep[i]);
  printf("\n");
  /* Calculate approximate error estimates which depends on the 1-norm)
   * of matrix T. The 1-norm of T is calculated using nag_dge_norm (f16rac).
   */
  nag_dge_norm(order, Nag_OneNorm, n, n, t, pdt, &tnorm, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dge_norm (f16rac).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* error estimates also depend on nag_machine_precision (x02ajc). */
  eps = nag_machine_precision;
  printf("\nApproximate error estimates for eigenvalues"
          "of T (machine dependent)\n");
  for (i = 0; i < m; ++i)
    printf("%11.1e", eps*tnorm/s[i]);
  printf("\n\nApproximate error estimates for right eigenvectors"
          "of T (machine dependent)\n");
  for (i = 0; i < m; ++i)
    printf("%11.1e", eps*tnorm/sep[i]);
  printf("\n");
 END:
  NAG_FREE(t);
  NAG_FREE(s);
  NAG_FREE(sep);
  NAG_FREE(vl);
  NAG_FREE(vr);

  return exit_status;
}
```

10.2 Program Data

```
nag_dtrsna (f08qlc) Example Program Data
  4                                   :Value of N
  0.7995  -0.1144   0.0060   0.0336
  0.0000  -0.0994   0.2478   0.3474
  0.0000  -0.6483  -0.0994   0.2026
  0.0000   0.0000   0.0000  -0.1007   :End of matrix T
```

10.3 Program Results

```
nag_dtrsna (f08qlc) Example Program Results

S
    9.9e-01    7.0e-01    7.0e-01    5.7e-01

Sep
    6.3e-01    3.7e-01    3.7e-01    3.1e-01

Approximate error estimates for eigenvaluesof T (machine dependent)
    9.6e-17    1.4e-16    1.4e-16    1.7e-16

Approximate error estimates for right eigenvectorsof T (machine dependent)
    1.5e-16    2.6e-16    2.6e-16    3.1e-16
```