

# NAG Library Function Document

## nag\_dormtr (f08fgc)

### 1 Purpose

nag\_dormtr (f08fgc) multiplies an arbitrary real matrix  $C$  by the real orthogonal matrix  $Q$  which was determined by nag\_dsytrd (f08fec) when reducing a real symmetric matrix to tridiagonal form.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_dormtr (Nag_OrderType order, Nag_SideType side, Nag_UploType uplo,
                Nag_TransType trans, Integer m, Integer n, const double a[],
                Integer pda, const double tau[], double c[], Integer pdc,
                NagError *fail)
```

### 3 Description

nag\_dormtr (f08fgc) is intended to be used after a call to nag\_dsytrd (f08fec), which reduces a real symmetric matrix  $A$  to symmetric tridiagonal form  $T$  by an orthogonal similarity transformation:  $A = QTQ^T$ . nag\_dsytrd (f08fec) represents the orthogonal matrix  $Q$  as a product of elementary reflectors.

This function may be used to form one of the matrix products

$$QC, Q^T C, CQ \text{ or } CQ^T,$$

overwriting the result on  $C$  (which may be any real rectangular matrix).

A common application of this function is to transform a matrix  $Z$  of eigenvectors of  $T$  to the matrix  $QZ$  of eigenvectors of  $A$ .

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **side** – Nag\_SideType *Input*  
*On entry:* indicates how  $Q$  or  $Q^T$  is to be applied to  $C$ .  
**side** = Nag\_LeftSide  
 $Q$  or  $Q^T$  is applied to  $C$  from the left.

- side** = Nag\_RightSide  
 $Q$  or  $Q^T$  is applied to  $C$  from the right.  
*Constraint:* **side** = Nag\_LeftSide or Nag\_RightSide.
- 3: **uplo** – Nag\_UploType *Input*  
*On entry:* this **must** be the same argument **uplo** as supplied to nag\_dsytrd (f08fec).  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.
- 4: **trans** – Nag\_TransType *Input*  
*On entry:* indicates whether  $Q$  or  $Q^T$  is to be applied to  $C$ .  
**trans** = Nag\_NoTrans  
 $Q$  is applied to  $C$ .  
**trans** = Nag\_Trans  
 $Q^T$  is applied to  $C$ .  
*Constraint:* **trans** = Nag\_NoTrans or Nag\_Trans.
- 5: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $C$ ;  $m$  is also the order of  $Q$  if **side** = Nag\_LeftSide.  
*Constraint:* **m**  $\geq$  0.
- 6: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $C$ ;  $n$  is also the order of  $Q$  if **side** = Nag\_RightSide.  
*Constraint:* **n**  $\geq$  0.
- 7: **a**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **a** must be at least  
 $\max(1, \mathbf{pda} \times \mathbf{m})$  when **side** = Nag\_LeftSide;  
 $\max(1, \mathbf{pda} \times \mathbf{n})$  when **side** = Nag\_RightSide.  
*On entry:* details of the vectors which define the elementary reflectors, as returned by nag\_dsytrd (f08fec).
- 8: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix  $A$  in the array **a**.  
*Constraints:*  
if **side** = Nag\_LeftSide, **pda**  $\geq$   $\max(1, \mathbf{m})$ ;  
if **side** = Nag\_RightSide, **pda**  $\geq$   $\max(1, \mathbf{n})$ .
- 9: **tau**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **tau** must be at least  
 $\max(1, \mathbf{m} - 1)$  when **side** = Nag\_LeftSide;  
 $\max(1, \mathbf{n} - 1)$  when **side** = Nag\_RightSide.  
*On entry:* further details of the elementary reflectors, as returned by nag\_dsytrd (f08fec).

10: **c**[*dim*] – double Input/Output

**Note:** the dimension, *dim*, of the array **c** must be at least

$\max(1, \mathbf{pdc} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pdc})$  when **order** = Nag\_RowMajor.

The (*i*, *j*)th element of the matrix *C* is stored in

**c**[(*j* – 1) × **pdc** + *i* – 1] when **order** = Nag\_ColMajor;  
**c**[(*i* – 1) × **pdc** + *j* – 1] when **order** = Nag\_RowMajor.

On entry: the *m* by *n* matrix *C*.

On exit: **c** is overwritten by *QC* or  $Q^T C$  or *CQ* or  $CQ^T$  as specified by **side** and **trans**.

11: **pdc** – Integer Input

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **c**.

Constraints:

if **order** = Nag\_ColMajor, **pdc** ≥ max(1, **m**);  
 if **order** = Nag\_RowMajor, **pdc** ≥ max(1, **n**).

12: **fail** – NagError \* Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument *<value>* had an illegal value.

### NE\_ENUM\_INT\_3

On entry, **side** = *<value>*, **m** = *<value>*, **n** = *<value>* and **pda** = *<value>*.

Constraint: if **side** = Nag\_LeftSide, **pda** ≥ max(1, **m**);

if **side** = Nag\_RightSide, **pda** ≥ max(1, **n**).

### NE\_INT

On entry, **m** = *<value>*.

Constraint: **m** ≥ 0.

On entry, **n** = *<value>*.

Constraint: **n** ≥ 0.

On entry, **pda** = *<value>*.

Constraint: **pda** > 0.

On entry, **pdc** = *<value>*.

Constraint: **pdc** > 0.

### NE\_INT\_2

On entry, **pdc** = *<value>* and **m** = *<value>*.

Constraint: **pdc** ≥ max(1, **m**).

On entry, **pdc** = *<value>* and **n** = *<value>*.

Constraint: **pdc** ≥ max(1, **n**).

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**7 Accuracy**

The computed result differs from the exact result by a matrix  $E$  such that

$$\|E\|_2 = O(\epsilon)\|C\|_2,$$

where  $\epsilon$  is the *machine precision*.

**8 Parallelism and Performance**

nag\_dormtr (f08fgc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dormtr (f08fgc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The total number of floating-point operations is approximately  $2m^2n$  if **side** = Nag\_LeftSide and  $2mn^2$  if **side** = Nag\_RightSide.

The complex analogue of this function is nag\_zunmtr (f08fuc).

**10 Example**

This example computes the two smallest eigenvalues, and the associated eigenvectors, of the matrix  $A$ , where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here  $A$  is symmetric and must first be reduced to tridiagonal form  $T$  by nag\_dsytrd (f08fec). The program then calls nag\_dstebz (f08jjc) to compute the requested eigenvalues and nag\_dstein (f08jkc) to compute the associated eigenvectors of  $T$ . Finally nag\_dormtr (f08fgc) is called to transform the eigenvectors to those of  $A$ .

**10.1 Program Text**

```

/* nag_dormtr (f08fgc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagx04.h>

int main(void)
{

```

```

/* Scalars */
Integer      i, j, m, n, nsplit, pda, pdz, d_len, e_len, tau_len;
Integer      exit_status = 0;
double       vl = 0.0, vu = 0.0;
NagError     fail;
Nag_UploType uplo;
Nag_OrderType order;
/* Arrays */
char         nag_enum_arg[40];
Integer      *iblock = 0, *ifailv = 0, *isplit = 0;
double       *a = 0, *d = 0, *e = 0, *tau = 0, *w = 0, *z = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J - 1) * pda + I - 1]
#define Z(I, J) z[(J - 1) * pdz + I - 1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I - 1) * pda + J - 1]
#define Z(I, J) z[(I - 1) * pdz + J - 1]
  order = Nag_RowMajor;
#endif

INIT_FAIL(fail);

printf("nag_dormtr (f08fgc) Example Program Results\n\n");

/* Skip heading in data file */
scanf("%*[\n] ");
scanf("%ld%*[\n] ", &n);
pda = n;
pdz = n;

tau_len = n-1;
d_len = n;
e_len = n-1;
/* Allocate memory */
if (!(a = NAG_ALLOC(n * n, double)) ||
    !(d = NAG_ALLOC(d_len, double)) ||
    !(e = NAG_ALLOC(e_len, double)) ||
    !(iblock = NAG_ALLOC(n, Integer)) ||
    !(ifailv = NAG_ALLOC(n, Integer)) ||
    !(isplit = NAG_ALLOC(n, Integer)) ||
    !(w = NAG_ALLOC(n, double)) ||
    !(tau = NAG_ALLOC(tau_len, double)) ||
    !(z = NAG_ALLOC(n * n, double)))
{
  printf("Allocation failure\n");
  exit_status = -1;
  goto END;
}

/* Read A from data file */
scanf("%39s%*[\n] ", nag_enum_arg);
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
if (uplo == Nag_Upper)
{
  for (i = 1; i <= n; ++i)
  {
    for (j = i; j <= n; ++j)
      scanf("%lf", &A(i, j));
  }
  scanf("%*[\n] ");
}
else
{
  for (i = 1; i <= n; ++i)
  {
    for (j = 1; j <= i; ++j)

```

```

        scanf("%lf", &A(i, j));
    }
    scanf("%*[\n] ");
}

/* Reduce A to tridiagonal form T = (Q**T)*A*Q */
/* nag_dsytrd (f08fec).
 * Orthogonal reduction of real symmetric matrix to
 * symmetric tridiagonal form
 */
nag_dsytrd(order, uplo, n, a, pda, d, e, tau, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dsytrd (f08fec).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Calculate the two smallest eigenvalues of T (same as A) */
/* nag_dstebz (f08jjc).
 * Selected eigenvalues of real symmetric tridiagonal matrix
 * by bisection
 */
nag_dstebz(Nag_Indices, Nag_ByBlock, n, vl, vu, 1, 2, 0.0,
           d, e, &m, &nsplit, w, iblock, isplit, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dstebz (f08jjc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print eigenvalues */
printf("Eigenvalues\n");
for (i = 0; i < m; ++i)
    printf("%8.4f%s", w[i], (i+1)%8 == 0?"\n":"" );
printf("\n\n");
/* Calculate the eigenvectors of T storing the result in Z */
/* nag_dstein (f08jkc).
 * Selected eigenvectors of real symmetric tridiagonal
 * matrix by inverse iteration, storing eigenvectors in real
 * array
 */
nag_dstein(order, n, d, e, m, w, iblock, isplit, z, pdz, ifailv,
           &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dstein (f08jkc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Calculate all the eigenvectors of A = Q*(eigenvectors of T) */
/* nag_dormtr (f08fgc).
 * Apply orthogonal transformation determined by nag_dsytrd
 * (f08fec)
 */
nag_dormtr(order, Nag_LeftSide, uplo, Nag_NoTrans, n, m, a, pda,
           tau, z, pdz, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dormtr (f08fgc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Normalize the eigenvectors */
for(j=1; j<=m; j++)
{
    for(i=n; i>=1; i--)
    {
        Z(i, j) = Z(i, j) / Z(1,j);
    }
}

```

```

/* Print eigenvectors */
/* nag_gen_real_mat_print (x04cac).
 * Print real general matrix (easy-to-use)
 */
fflush(stdout);
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, m, z,
                      pdz, "Eigenvectors", 0, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
          fail.message);
    exit_status = 1;
    goto END;
}
END:
NAG_FREE(a);
NAG_FREE(d);
NAG_FREE(e);
NAG_FREE(iblock);
NAG_FREE(ifailv);
NAG_FREE(isplit);
NAG_FREE(tau);
NAG_FREE(w);
NAG_FREE(z);

return exit_status;
}

```

## 10.2 Program Data

```

nag_dormtr (f08fgc) Example Program Data
  4                               :Value of n
  Nag_Lower                       :Value of uplo
  2.07
  3.87  -0.21
  4.20  1.87  1.15
  -1.15  0.63  2.06  -1.81  :End of matrix A

```

## 10.3 Program Results

```

nag_dormtr (f08fgc) Example Program Results

```

```

Eigenvalues
-5.0034  -1.9987

```

```

Eigenvectors
      1      2
1      1.0000      1.0000
2     -0.6148     -3.4333
3     -0.8378      1.7553
4      1.0219     -1.6052

```

---