

# NAG Library Function Document

## nag\_dtpcon (f07ugc)

### 1 Purpose

nag\_dtpcon (f07ugc) estimates the condition number of a real triangular matrix, using packed storage.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dtpcon (Nag_OrderType order, Nag_NormType norm, Nag_UploType uplo,
                Nag_DiagType diag, Integer n, const double ap[], double *rcond,
                NagError *fail)
```

### 3 Description

nag\_dtpcon (f07ugc) estimates the condition number of a real triangular matrix  $A$ , in either the 1-norm or the  $\infty$ -norm, using packed storage:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that  $\kappa_\infty(A) = \kappa_1(A^T)$ .

Because the condition number is infinite if  $A$  is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes  $\|A\|_1$  or  $\|A\|_\infty$  exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate  $\|A^{-1}\|_1$  or  $\|A^{-1}\|_\infty$ .

### 4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **norm** – Nag\_NormType *Input*

*On entry:* indicates whether  $\kappa_1(A)$  or  $\kappa_\infty(A)$  is estimated.

**norm** = Nag\_OneNorm  
 $\kappa_1(A)$  is estimated.

**norm** = Nag\_InfNorm  
 $\kappa_\infty(A)$  is estimated.

*Constraint:* **norm** = Nag\_OneNorm or Nag\_InfNorm.

- 3: **uplo** – Nag\_UploType *Input*  
*On entry:* specifies whether  $A$  is upper or lower triangular.  
**uplo** = Nag\_Upper  
 $A$  is upper triangular.  
**uplo** = Nag\_Lower  
 $A$  is lower triangular.  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.
- 4: **diag** – Nag\_DiagType *Input*  
*On entry:* indicates whether  $A$  is a nonunit or unit triangular matrix.  
**diag** = Nag\_NonUnitDiag  
 $A$  is a nonunit triangular matrix.  
**diag** = Nag\_UnitDiag  
 $A$  is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.  
*Constraint:* **diag** = Nag\_NonUnitDiag or Nag\_UnitDiag.
- 5: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 6: **ap**[*dim*] – const double *Input*  
**Note:** the dimension, *dim*, of the array **ap** must be at least  $\max(1, n \times (n + 1)/2)$ .  
*On entry:* the  $n$  by  $n$  triangular matrix  $A$ , packed by rows or columns.  
The storage of elements  $A_{ij}$  depends on the **order** and **uplo** arguments as follows:  
if **order** = 'Nag\_ColMajor' and **uplo** = 'Nag\_Upper',  
 $A_{ij}$  is stored in **ap**[( $j - 1$ )  $\times$   $j/2 + i - 1$ ], for  $i \leq j$ ;  
if **order** = 'Nag\_ColMajor' and **uplo** = 'Nag\_Lower',  
 $A_{ij}$  is stored in **ap**[( $2n - j$ )  $\times$  ( $j - 1$ )/2 +  $i - 1$ ], for  $i \geq j$ ;  
if **order** = 'Nag\_RowMajor' and **uplo** = 'Nag\_Upper',  
 $A_{ij}$  is stored in **ap**[( $2n - i$ )  $\times$  ( $i - 1$ )/2 +  $j - 1$ ], for  $i \leq j$ ;  
if **order** = 'Nag\_RowMajor' and **uplo** = 'Nag\_Lower',  
 $A_{ij}$  is stored in **ap**[( $i - 1$ )  $\times$   $i/2 + j - 1$ ], for  $i \geq j$ .  
If **diag** = 'Nag\_UnitDiag', the diagonal elements of AP are assumed to be 1, and are not referenced; the same storage scheme is used whether **diag** = 'Nag\_NonUnitDiag' or **diag** = 'Nag\_UnitDiag'.
- 7: **rcond** – double \* *Output*  
*On exit:* an estimate of the reciprocal of the condition number of  $A$ . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*,  $A$  is singular to working precision.
- 8: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{n} = \langle value \rangle$ .  
Constraint:  $\mathbf{n} \geq 0$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

## 7 Accuracy

The computed estimate **rcond** is never less than the true value  $\rho$ , and in practice is nearly always less than  $10\rho$ , although examples can be constructed where **rcond** is much larger.

## 8 Parallelism and Performance

nag\_dtpcon (f07ugc) is not threaded by NAG in any implementation.

nag\_dtpcon (f07ugc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

A call to nag\_dtpcon (f07ugc) involves solving a number of systems of linear equations of the form  $Ax = b$  or  $A^T x = b$ ; the number is usually 4 or 5 and never more than 11. Each solution involves approximately  $n^2$  floating-point operations but takes considerably longer than a call to nag\_dtptrs (f07uec) with one right-hand side, because extra care is taken to avoid overflow when  $A$  is approximately singular.

The complex analogue of this function is nag\_ztpcon (f07uuc).

## 10 Example

This example estimates the condition number in the 1-norm of the matrix  $A$ , where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix},$$

using packed storage. The true condition number in the 1-norm is 116.41.

## 10.1 Program Text

```

/* nag_dtpcon (f07ugc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double      rcond;
    Integer     ap_len, i, j, n;
    Integer     exit_status = 0;
    Nag_UploType uplo;
    NagError    fail;
    Nag_OrderType order;
    /* Arrays */
    char        nag_enum_arg[40];
    double      *ap = 0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
    order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dtpcon (f07ugc) Example Program Results\n");
    /* Skip heading in data file */
    scanf("%*[\n] ");
    scanf("%ld%*[\n] ", &n);

    /* Allocate memory */
    ap_len = n*(n+1)/2;
    if (!(ap = NAG_ALLOC(ap_len, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
    scanf(" %39s%*[\n] ", nag_enum_arg);
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    if (uplo == Nag_Upper)
    {
        for (i = 1; i <= n; ++i)
        {
            for (j = i; j <= n; ++j)
                scanf("%lf", &A_UPPER(i, j));
        }
        scanf("%*[\n] ");
    }
    else

```

```

    {
        for (i = 1; i <= n; ++i)
        {
            for (j = 1; j <= i; ++j)
                scanf("%lf", &A_LOWER(i, j));
        }
        scanf("%*[\n] ");
    }

/* Estimate condition number */
/* nag_dtpcon (f07ugc).
 * Estimate condition number of real triangular matrix,
 * packed storage
 */
nag_dtpcon(order, Nag_OneNorm, uplo, Nag_NonUnitDiag, n,
           ap, &rcond, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dtpcon (f07ugc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
/* nag_machine_precision (x02ajc).
 * The machine precision
 */
if (rcond >= nag_machine_precision)
    printf("Estimate of condition number =%11.2e\n", 1.0 / rcond);
else
    printf("A is singular to working precision\n");
END:
NAG_FREE(ap);

return exit_status;
}

```

## 10.2 Program Data

```

nag_dtpcon (f07ugc) Example Program Data
4                               :Value of n
Nag_Lower                       :Value of uplo
4.30
-3.96  -4.87
0.40   0.31  -8.02
-0.27  0.07  -5.95  0.12   :End of matrix A

```

## 10.3 Program Results

```

nag_dtpcon (f07ugc) Example Program Results

Estimate of condition number = 1.16e+02

```

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