NAG Library Function Document

nag_dsptri (f07pjc)

1 Purpose

nag_dsptri (f07pjc) computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by nag_dsptrf (f07pdc), using packed storage.

2 Specification

```
#include <nag.h>
#include <nagf07.h>
```

```
void nag_dsptri (Nag_OrderType order, Nag_UploType uplo, Integer n,
     double ap[], const Integer ipiv[], NagError *fail)
```

3 Description

nag_dsptri (f07pjc) is used to compute the inverse of a real symmetric indefinite matrix A, the function must be preceded by a call to nag_dsptrf (f07pdc), which computes the Bunch–Kaufman factorization of A, using packed storage.

If **uplo** $=$ Nag_Upper, $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{T}}P^{\mathrm{T}}XPU = D^{-1}$.

If **uplo** $=$ Nag_Lower, $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{T}}P^{\mathrm{T}}XPL = D^{-1}$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** $=$ Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** $=$ Nag_RowMajor or Nag_ColMajor.

2: **uplo** – Nag_UploType *Input*

On entry: specifies how A has been factorized.

uplo $=$ Nag_Upper
$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo $=$ Nag_Lower
$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** $=$ Nag_Upper or Nag_Lower.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **ap**[dim] – double *Input/Output*

Note: the dimension, *dim*, of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

On entry: the factorization of A stored in packed form, as returned by nag_dsptrf (f07pdc).

On exit: the factorization is overwritten by the n by n matrix A^{-1}.

The storage of elements A_{ij} depends on the **order** and **uplo** arguments as follows:

if **order** = 'Nag_ColMajor' and **uplo** = 'Nag_Upper',
  A_{ij} is stored in **ap**$[(j-1) \times j/2 + i - 1]$, for $i \leq j$;
if **order** = 'Nag_ColMajor' and **uplo** = 'Nag_Lower',
  A_{ij} is stored in **ap**$[(2n-j) \times (j-1)/2 + i - 1]$, for $i \geq j$;
if **order** = 'Nag_RowMajor' and **uplo** = 'Nag_Upper',
  A_{ij} is stored in **ap**$[(2n-i) \times (i-1)/2 + j - 1]$, for $i \leq j$;
if **order** = 'Nag_RowMajor' and **uplo** = 'Nag_Lower',
  A_{ij} is stored in **ap**$[(i-1) \times i/2 + j - 1]$, for $i \geq j$.

5: **ipiv**[dim] – const Integer *Input*

Note: the dimension, *dim*, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

On entry: details of the interchanges and the block structure of D, as returned by nag_dsptrf (f07pdc).

6: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_SINGULAR

$d(\langle value \rangle, \langle value \rangle)$ is exactly zero. D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if **uplo** = Nag_Upper, $|DU^{\mathrm{T}}P^{\mathrm{T}}XPU - I| \leq c(n)\epsilon\left(|D||U^{\mathrm{T}}|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;

if **uplo** = Nag_Lower, $|DL^{\mathrm{T}}P^{\mathrm{T}}XPL - I| \leq c(n)\epsilon\left(|D||L^{\mathrm{T}}|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Parallelism and Performance

nag_dsptri (f07pjc) is not threaded by NAG in any implementation.

nag_dsptri (f07pjc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this function are nag_zhptri (f07pwc) for Hermitian matrices and nag_zsptri (f07qwc) for symmetric matrices.

10 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite, stored in packed form, and must first be factorized by nag_dsptrf (f07pdc).

10.1 Program Text

```
/* nag_dsptri (f07pjc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 * Mark 7b revised, 2004.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer       i, j, n;
  Integer       exit_status = 0;
  NagError      fail;
  Nag_UploType  uplo;
  Nag_OrderType order;
  /* Arrays */
  Integer       *ipiv = 0;
  char          nag_enum_arg[40];
  double        *ap = 0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
  order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
  order = Nag_RowMajor;
```

```
#endif

  INIT_FAIL(fail);

  printf("nag_dsptri (f07pjc) Example Program Results\n\n");

  /* Skip heading in data file */
  scanf("%*[^\n] ");
  scanf("%ld%*[^\n] ", &n);

  /* Allocate memory */
  if (!(ipiv = NAG_ALLOC(n, Integer)) ||
      !(ap = NAG_ALLOC(n * (n + 1)/2, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  scanf(" %39s%*[^\n] ", nag_enum_arg);
  /* nag_enum_name_to_value (x04nac).
   * Converts NAG enum member name to value
   */
  uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

  if (uplo == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
            scanf("%lf", &A_UPPER(i, j));
        }
      scanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
            scanf("%lf", &A_LOWER(i, j));
        }
      scanf("%*[^\n] ");
    }

  /* Factorize A */
  /* nag_dsptrf (f07pdc).
   * Bunch-Kaufman factorization of real symmetric indefinite
   * matrix, packed storage
   */
  nag_dsptrf(order, uplo, n, ap, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dsptrf (f07pdc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute inverse of A */
  /* nag_dsptri (f07pjc).
   * Inverse of real symmetric indefinite matrix, matrix
   * already factorized by nag_dsptrf (f07pdc), packed storage
   */
  nag_dsptri(order, uplo, n, ap, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dsptri (f07pjc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print inverse */
  /* nag_pack_real_mat_print (x04ccc).
```

```
   * Print real packed triangular matrix (easy-to-use)
   */
  fflush(stdout);
  nag_pack_real_mat_print(order, uplo, Nag_NonUnitDiag, n, ap,
                          "Inverse", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_pack_real_mat_print (x04ccc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  NAG_FREE(ipiv);
  NAG_FREE(ap);
  return exit_status;
}
```

10.2 Program Data

```
nag_dsptri (f07pjc) Example Program Data
  4                          :Value of n
  Nag_Lower                  :Value of uplo
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

10.3 Program Results

```
nag_dsptri (f07pjc) Example Program Results

 Inverse
           1          2          3          4
 1      0.7485
 2      0.5221    -0.1605
 3     -1.0058    -0.3131     1.3501
 4     -1.4386    -0.7440     2.0667     2.4547
```