NAG Library Function Document

nag_dpbtrf (f07hdc)

1 Purpose

nag_dpbtrf (f07hdc) computes the Cholesky factorization of a real symmetric positive definite band matrix.

2 Specification

```
#include <nag.h>
#include <nagf07.h>
void nag_dpbtrf (Nag_OrderType order, Nag_UploType uplo, Integer n,
     Integer kd, double ab[], Integer pdab, NagError *fail)
```

3 Description

nag_dpbtrf (f07hdc) forms the Cholesky factorization of a real symmetric positive definite band matrix A either as $A = U^{\mathrm{T}}U$ if **uplo** = Nag_Upper or $A = LL^{\mathrm{T}}$ if **uplo** = Nag_Lower, where U (or L) is an upper (or lower) triangular band matrix with the same number of superdiagonals (or subdiagonals) as A.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **uplo** – Nag_UploType *Input*

On entry: specifies whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = Nag_Upper
The upper triangular part of A is stored and A is factorized as $U^{\mathrm{T}}U$, where U is upper triangular.

uplo = Nag_Lower
The lower triangular part of A is stored and A is factorized as LL^{T}, where L is lower triangular.

Constraint: **uplo** = Nag_Upper or Nag_Lower.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **kd** – Integer *Input*

On entry: k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

5: **ab**[dim] – double *Input/Output*

Note: the dimension, *dim*, of the array **ab** must be at least $\max(1, \textbf{pdab} \times \textbf{n})$.

On entry: the n by n symmetric band matrix A.

This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of A_{ij}, depends on the **order** and **uplo** arguments as follows:

if **order** = 'Nag_ColMajor' and **uplo** = 'Nag_Upper',
A_{ij} is stored in **ab**$[k_d + i - j + (j-1) \times \textbf{pdab}]$, for $j = 1, \ldots, n$ and $i = \max(1, j - k_d), \ldots, j$;
if **order** = 'Nag_ColMajor' and **uplo** = 'Nag_Lower',
A_{ij} is stored in **ab**$[i - j + (j-1) \times \textbf{pdab}]$, for $j = 1, \ldots, n$ and $i = j, \ldots, \min(n, j + k_d)$;
if **order** = 'Nag_RowMajor' and **uplo** = 'Nag_Upper',
A_{ij} is stored in **ab**$[j - i + (i-1) \times \textbf{pdab}]$, for $i = 1, \ldots, n$ and $j = i, \ldots, \min(n, i + k_d)$;
if **order** = 'Nag_RowMajor' and **uplo** = 'Nag_Lower',
A_{ij} is stored in **ab**$[k_d + j - i + (i-1) \times \textbf{pdab}]$, for $i = 1, \ldots, n$ and $j = \max(1, i - k_d), \ldots, i$.

On exit: the upper or lower triangle of A is overwritten by the Cholesky factor U or L as specified by **uplo**, using the same storage format as described above.

6: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **ab**.

Constraint: **pdab** $\geq$ **kd** $+ 1$.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **kd** = ⟨*value*⟩.
Constraint: **kd** ≥ 0.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **pdab** = ⟨*value*⟩.
Constraint: **pdab** > 0.

NE_INT_2

On entry, **pdab** $= \langle value \rangle$ and **kd** $= \langle value \rangle$.
Constraint: **pdab** $\geq$ **kd** $+ 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_POS_DEF

The leading minor of order $\langle value \rangle$ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. There is no function specifically designed to factorize a symmetric band matrix which is not positive definite; the matrix must be treated either as a nonsymmetric band matrix, by calling nag_dgbtrf (f07bdc) or as a full symmetric matrix, by calling nag_dsytrf (f07mdc).

7 Accuracy

If **uplo** $=$ Nag_Upper, the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(k+1)\epsilon \left|U^{\mathrm{T}}\right||U|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If **uplo** $=$ Nag_Lower, a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(k+1)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Parallelism and Performance

nag_dpbtrf (f07hdc) is not threaded by NAG in any implementation.

nag_dpbtrf (f07hdc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $n(k+1)^2$, assuming $n \gg k$.

A call to nag_dpbtrf (f07hdc) may be followed by calls to the functions:

nag_dpbtrs (f07hec) to solve $AX = B$;

nag_dpbcon (f07hgc) to estimate the condition number of A.

The complex analogue of this function is nag_zpbtrf (f07hrc).

10 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 5.49 & 2.68 & 0.00 & 0.00 \\ 2.68 & 5.63 & -2.39 & 0.00 \\ 0.00 & -2.39 & 2.60 & -2.22 \\ 0.00 & 0.00 & -2.22 & 5.17 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_dpbtrf (f07hdc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer       i, j, k, kd, n, pdab;
  Integer       exit_status = 0;
  Nag_UploType  uplo;
  NagError      fail;
  Nag_OrderType order;
  /* Arrays */
  char          nag_enum_arg[40];
  double        *ab = 0;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I, J) ab[(J-1)*pdab + k + I - J - 1]
#define AB_LOWER(I, J) ab[(J-1)*pdab + I - J]
  order = Nag_ColMajor;
#else
#define AB_UPPER(I, J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I, J) ab[(I-1)*pdab + k + J - I - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_dpbtrf (f07hdc) Example Program Results\n\n");

  /* Skip heading in data file */
  scanf("%*[^\n] ");
  scanf("%ld%ld%*[^\n] ", &n, &kd);
  pdab = kd + 1;

  /* Allocate memory */
  if (!(ab = NAG_ALLOC((kd+1) * n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  scanf(" %39s%*[^\n] ", nag_enum_arg);
  /* nag_enum_name_to_value (x04nac).
   * Converts NAG enum member name to value
   */
  uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

  k = kd + 1;
  if (uplo == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= MIN(i+kd, n); ++j)
            scanf("%lf", &AB_UPPER(i, j));
        }
      scanf("%*[^\n] ");
    }
```

```
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = MAX(1, i-kd); j <= i; ++j)
            scanf("%lf", &AB_LOWER(i, j));
        }
      scanf("%*[^\n] ");
    }
  /* Factorize A */
  /* nag_dpbtrf (f07hdc).
   * Cholesky factorization of real symmetric
   * positive-definite band matrix
   */
  fflush(stdout);
  nag_dpbtrf(order, uplo, n, kd, ab, pdab, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dpbtrf (f07hdc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print details of factorization */
  if (uplo == Nag_Upper)
    /* nag_band_real_mat_print (x04cec).
     * Print real packed banded matrix (easy-to-use)
     */
    nag_band_real_mat_print(order, n, n, 0, kd, ab, pdab, "Factor",
                            0, &fail);
  else
    /* nag_band_real_mat_print (x04cec), see above. */
    fflush(stdout);
    nag_band_real_mat_print(order, n, n, kd, 0, ab, pdab, "Factor",
                            0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_band_real_mat_print (x04cec).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  NAG_FREE(ab);
  return exit_status;
}
```

10.2 Program Data

```
nag_dpbtrf (f07hdc) Example Program Data
  4  1                        :Values of n and kd
  Nag_Lower                   :Value of uplo
  5.49
  2.68   5.63
        -2.39   2.60
               -2.22   5.17   :End of matrix A
```

10.3 Program Results

```
nag_dpbtrf (f07hdc) Example Program Results

 Factor
           1          2          3          4
 1      2.3431
 2      1.1438     2.0789
 3                -1.1497     1.1306
 4                           -1.9635     1.1465
```