

NAG Library Function Document

nag_dgttrf (f07cdc)

1 Purpose

nag_dgttrf (f07cdc) computes the LU factorization of a real n by n tridiagonal matrix A .

2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dgttrf (Integer n, double dl[], double d[], double du[],
                double du2[], Integer ipiv[], NagError *fail)
```

3 Description

nag_dgttrf (f07cdc) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

5 Arguments

- 1: **n** – Integer *Input*
On entry: n , the order of the matrix A .
Constraint: $n \geq 0$.
- 2: **dl**[*dim*] – double *Input/Output*
Note: the dimension, *dim*, of the array **dl** must be at least $\max(1, n - 1)$.
On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A .
On exit: is overwritten by the $(n - 1)$ multipliers that define the matrix L of the LU factorization of A .
- 3: **d**[*dim*] – double *Input/Output*
Note: the dimension, *dim*, of the array **d** must be at least $\max(1, n)$.
On entry: must contain the n diagonal elements of the matrix A .
On exit: is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A .
- 4: **du**[*dim*] – double *Input/Output*
Note: the dimension, *dim*, of the array **du** must be at least $\max(1, n - 1)$.

On entry: must contain the $(n - 1)$ superdiagonal elements of the matrix A .

On exit: is overwritten by the $(n - 1)$ elements of the first superdiagonal of U .

5: **du2[n - 2]** – double *Output*

On exit: contains the $(n - 2)$ elements of the second superdiagonal of U .

6: **ipiv[n]** – Integer *Output*

On exit: contains the n pivot indices that define the permutation matrix P . At the i th step, row i of the matrix was interchanged with row **ipiv**[$i - 1$]. **ipiv**[$i - 1$] will always be either i or $(i + 1)$, **ipiv**[$i - 1$] = i indicating that a row interchange was not performed.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: **n** ≥ 0 .

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_SINGULAR

$U(\langle value \rangle, \langle value \rangle)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = PLU,$$

where

$$\|E\|_{\infty} = O(\epsilon)\|A\|_{\infty}$$

and ϵ is the *machine precision*.

Following the use of this function, nag_dgtrtrs (f07cec) can be used to solve systems of equations $AX = B$ or $A^T X = B$, and nag_dgtcon (f07cgc) can be used to estimate the condition number of A .

8 Parallelism and Performance

Not applicable.

9 Further Comments

The total number of floating-point operations required to factorize the matrix A is proportional to n .

The complex analogue of this function is nag_zgttrf (f07crc).

10 Example

This example factorizes the tridiagonal matrix A given by

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix}.$$

10.1 Program Text

```

/* nag_dgttrf (f07cdc) Example Program.
*
* Copyright 2004 Numerical Algorithms Group.
*
* Mark 23, 2011
*/

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, n;

    /* Arrays */
    double *d = 0, *dl = 0, *du = 0, *du2 = 0;
    Integer *ipiv = 0;

    /* Nag Types */
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_dgttrf (f07cdc) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n]");
    scanf("%ld%*[\n]", &n);
    if (n < 0)
    {
        printf("Invalid n\n");
        exit_status = 1;
        goto END;
    }
    /* Allocate memory */
    if (!(d = NAG_ALLOC(n, double)) ||
        !(dl = NAG_ALLOC(n-1, double)) ||
        !(du = NAG_ALLOC(n-1, double)) ||
        !(du2 = NAG_ALLOC(n-2, double)) ||
        !(ipiv = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read the tridiagonal matrix A from data file */
    for (i = 0; i < n - 1; ++i) scanf("%lf", &du[i]);
    scanf("%*[\n]");
    for (i = 0; i < n; ++i) scanf("%lf", &d[i]);
    scanf("%*[\n]");
    for (i = 0; i < n - 1; ++i) scanf("%lf", &dl[i]);
    scanf("%*[\n]");

```

```

/* Factorize the tridiagonal matrix A using nag_dgttrf (f07cdc). */
nag_dgttrf(n, dl, d, du, du2, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dgttrf (f07cdc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Print details of the factorization */
printf("Details of factorization\n\n");
printf(" Second super-diagonal of U\n");
for (i = 0; i < n - 2; ++i) printf("%9.4f%s", du2[i], i%8 == 7?"\n":" ");
printf("\n\n First super-diagonal of U\n");
for (i = 0; i < n - 1; ++i) printf("%9.4f%s", du[i], i%8 == 7?"\n":" ");
printf("\n\n Main diagonal of U\n");
for (i = 0; i < n; ++i) printf("%9.4f%s", d[i], i%8 == 7?"\n":" ");

printf("\n\n Multipliers\n");
for (i = 0; i < n - 1; ++i) printf("%9.4f%s", dl[i], i%8 == 7?"\n":" ");
printf("\n\n Vector of interchanges\n");
for (i = 0; i < n; ++i) printf("%9ld%s", ipiv[i], i%5 == 4?"\n":" ");
printf("\n");
END:
NAG_FREE(d);
NAG_FREE(dl);
NAG_FREE(du);
NAG_FREE(du2);
NAG_FREE(ipiv);
return exit_status;
}

```

10.2 Program Data

```

nag_dgttrf (f07cdc) Example Program Data
5          : n
      2.1  -1.0   1.9   8.0
3.0  2.3  -5.0  -0.9   7.1
3.4  3.6   7.0  -6.0          : matrix A

```

10.3 Program Results

nag_dgttrf (f07cdc) Example Program Results

Details of factorization

```

Second super-diagonal of U
-1.0000   1.9000   8.0000

First super-diagonal of U
 2.3000  -5.0000  -0.9000   7.1000

Main diagonal of U
 3.4000   3.6000   7.0000  -6.0000  -1.0154

Multipliers
 0.8824   0.0196   0.1401  -0.0148

Vector of interchanges
 2         3         4         5         5

```
